7 Harmonization of tetrachordal scales

SCALES BASED ON tetrachords are found in the musics of a large part of the world. Although much of this music is primarily melodic and heterophonic, this is due neither to the intrinsic nature of tetrachords nor to the scales derived from them. Rather, it is a matter of style and tradition. Many, if not most, tetrachordal scales have harmonic implications even if these implications are contrary to the familiar rules of European tonal harmony.

The melodies of the ancient Greeks were accompanied by more or less independent voices, but polyphony and harmony in their traditional senses appear to have been absent. "A feeling for the triad," however, does appear in the later Greek musical fragments, but this may be a modern and not ancient perception (Winnington-Ingram 1936).

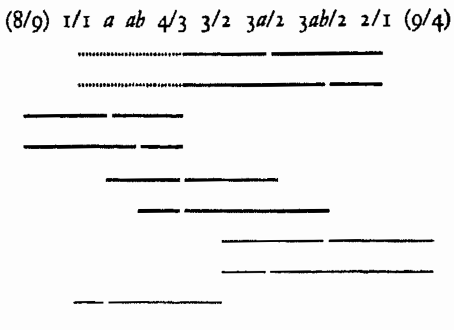
The scales of North Indian music are also based on tetrachords (Sachs 1943; Wilson 1986a, 1987). In this music, drones emphasizing the tonic and usually the dominant of the scale are essential elements of performance. Their function may be to fix the tonic so that ambiguous intervals are not exposed (chapter 5 and Rothenberg 1969, 1978).

Islamic music of the period of the great medieval theorists Al-Farabi, Safiyu-d-Din, and Avicenna (Ibn Sina) was likewise heterophonic rather than harmonic (Sachs 1943; D'Erlanger 1930, 1935, 1938). In recent times, however, some Islamic groups have adopted certain elements of tonal harmony into their music.

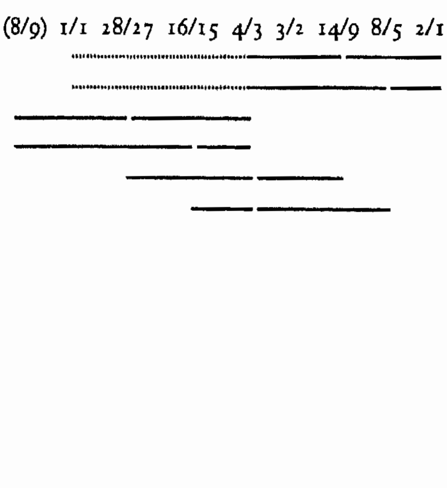
Harmonizing tetrachordal scales

Many tetrachordal scales are nevertheless suitable for harmonic music. The

7-1. Endogenous harmonization of tetrachordal scales. The addition of the subtonic 9/8 below 1/1 to the enharmonic and chromatic genera where it was called hyperhypate is attested both theoretically and musically (Winnington-Ingram 1936, 25). The dotted lines indicate the lower octave of the dominant of the triads on 4/3.



7-2. Endogenous harmonization of Archytas's enharmonic.



Lydian mode of Ptolemy's intense diatonic genus is the just intonation of the major mode. The diatonic Arabo-Persian scale *hhidjazi*, is more consonant than the 12-tone equal-tempered tuning of the major scale (Helmholtz [1877] 1954).

Harry Partch pointed out that many of the other tetrachordal genera also have harmonic implications which may be exploited in the context of extended just intonation (Partch [1949] 1974). As an example, he offered Wilfrid Perrett's harmonization of a version of the enharmonic tetrachord. Partch added a repeat to Perrett's progression and transposed it into his 43-tone scale (Partch [1949] 1974; Perrett 1926).

Partch also challenged his readers to limit themselves to the notes of the scale. 7-1 depicts the triadic resources of a generalized tetrachordal scale in which both tetrachords are identical. The dark lines delimit triads which are available in all genera while the light ones indicate chords which may or may not be consonant in certain genera.

The three sub-intervals of the tetrachord are denoted as *a*, *b*, and 4/3*ab*, resulting in the tones, 1/1, *a*, *ab*, and 4/3, duplicated on the 3/2. Because there is both musical and literary evidence for the customary addition of the note hyperhypate a 9/8 whole tone below the tonic in the enharmonic and chromatic genera (Winnington-Ingram 1936, 25), it has been included. The inversion of this interval has also been added to allow the construction of a consonant dominant triad in some genera or permutations.

The types of these triads depend upon the tuning of the tetrachord. In Archytas's enharmonic genus, the triads on 4/3 and 8/9 will be septimal minor, 6:7:9. The triad on *a* (28/27) is the septimal major triad, 14:18:21. The triad on *ab* (16/15) is a major triad, 4:5:6, and the alternative triads on 4/3 and 8/9, are minor, 10:12:15. The tonal center appears not to be the 1/1, but rather the 4/3 or mese. These chords are shown in 7-2.

The tonal functions of these triads are determined by the mode or circular permutation of the scale. The Lydian or C mode of Ptolemy's intense diatonic, in its normal form, 16/15 · 9/8 · 10/9, is the familiar major mode with 4:5:6 triads on 1/1, 4/3, and 3/2. The reverse arrangement of this tetrachord, 10/9 · 9/8 · 16/15, generates the natural minor mode with 10:12:15 or subharmonic 4:5:6 triads on these degrees. This scale is not identical to the Hypodorian or A mode of the first scale because that scale has a 27/20 rather than a 4/3 as its fourth degree. The chordal matrices and tetrachordal forms of these scales are shown in 7-3.

7-3. The 4:5:6 triad and its derived tritriadic scale. The tritriadic or matrix form is the C or Lydian mode of the tetrachordal scale. The tonic of the triad is denoted t *or 1/1, the third or mediant,* m *and the fifth or dominant,* d. *The tetrachordal form is the E or Dorian mode of the tritriadic scale.*

SUBDOMINANT	4/3 5/3 2/1	*2/d m/d 2/1*
TONIC	1/1 5/4 3/2	*1/1 m d*
DOMINANT	3/2 15/8 9/8	*d d·m* d^2

1/1 9/8 5/4 4/3 3/2 5/3 15/8 2/1

9/8 · 10/9 · 16/15 · 9/8 · 10/9 · 9/8 · 16/15

THE TETRACHORDAL FORM

1/1 16/15 6/5 4/3 3/2 8/5 9/5 2/1

16/15 · 9/8 · 10/9 · 9/8 · 16/15 · 9/8 · 10/9

(16/15 · 9/8 · 10/9)

THE 10:12:15 TRIAD & ITS DERIVED TRITRIADIC SCALE

SUBDOMINANT	4/3 8/5 2/1	*2/d m/d 2/1*
TONIC	1/1 6/5 3/2	*1/1 m d*
DOMINANT	3/2 9/5 9/8	*d d·m* d^2

1/1 9/8 6/5 4/3 3/2 8/5 9/5 2/1

9/8 · 16/15 · 10/9 · 9/8 · 16/15 · 9/8 · 10/9

THE TETRACHORDAL FORM

1/1 10/9 5/4 4/3 3/2 5/3 15/8 2/1

10/9 · 9/8 · 16/15 · 9/8 · 10/9 · 9/8 · 16/5

(10/9 · 9/8 · 16/15)

The seven modes or octave species of the reversed tetrachord scale are the exact inversions of those of the major scale above. The C mode of this scale is the diatonic scale of John Redfield (1928, 191–197). Redfield assigned Hebraic names to these modes and termed the triads with the comma-enlarged fifth "Doric."

The mode that is the inversion of the major scale may be harmonized with three triads built downwards from 2/1, 3/2, and 4/3. An otherwise obscure composer named Blainville wrote a short symphony in this scale and was ridiculed by Rousseau for doing so (Perrett 1931; Partch [1949] 1974). This kind of inverted harmony was called the *phonic system* by the nineteenth and early twentieth century theorist von Öttingen (Helmholtz [1877] 1954; Mandelbaum 1961) in contrast to the traditional *tonic* system.

Tritriadic scales

The scales derived from tetrachords with 9/8 as their second interval may be called *tritriadics* because they may be divided into three triads on the roots 1/1, 4/3, and 3/2. They are harmonizable with analogs of the familiar I IV (I) V I and I IV (VII) III VI (II) V I progressions (Chalmers 1979, 1986, 1987, 1988).

In general, however, the VII and II chords will be out of tune (Lewin 1982) and probably should be omitted in the progressions unless extra notes are employed. The composer Erling Wold, however, has made a case for a more adventurous utilization of available tonal resources (Wold 1988). Partch ([1949] 1974) has done so too in a discussion of a letter from Fox-Strangways concerning the alleged defects of just intonation and their effect on modulation.

The three primary triads on 1/1, 4/3, and 3/2 are of the same type, but the triads on the third (mediant) and sixth (submediant) degrees are of the conjugate or 3/2's complement type. For example, the primary triads of number 1a of 7-4 are major, while the mediant and submediant triads are minor. In number 1b, the modalities are just the reverse. In addition to the principle triads of these scales, triads on other degrees may also be usable. Similarly, in some tunings, seventh or other chords may be useful.

Phonic or descending harmonizations are also possible in certain modes of tritriadic scales. Lewin, in fact, proposes what might be called both phonic major and minor harmonizations (Lewin 1982).

The generalized triad is denoted as *t:m:d*, after Lewin (1982), where *t* is the tonic, *m* the mediant, and *d* the dominant. In principle, any tetrachord containing the interval 9/8 can be arranged as a tritriadic generator, but the majority of the resulting triads will be relatively discordant. If the mediant of a triad is denoted by *m*, then the tetrachord has the form $4/3m \cdot 9/8 \cdot 8m/9$, where $4/3m \cdot 8m/9 = 32/27$. The conjugate tritriadic scale is generated by the permutation $8m/9 \cdot 9/8 \cdot 4/3m$. The magnitude of *m* may range from 9/8 to 4/3 and generate a seven tone tritriadic scale, though the Rothenberg propriety (chapter 5) of the scale and the consonance of the triads will depend of the value of *m*.

Triads with perfect fifths ($d = 3/2$) whose mediants (*m*) are greater than 32/27 and less than 81/64 generate strictly proper scales (chapter 5; Rothenberg 1969, 1975, 1978; Chalmers 1975). Strictly proper scales tend to be perceived as musical gestalts and are used in styles where motivic transposition is an important structural element. Improper scales, on the other hand, are usually employed as sets of principal and auxiliary or ornamental tones.

Only a limited number of acceptably consonant triads exist in just intonation and also generate useful tritriadic scales. The most important of these have been tabulated in 7-4. As indicated above, triads 1a and 1b generate the major and natural minor modes, and 2a and 2b generate the

7-4. *Tritriadic tetrachords. I stands for "improper," and SP for "strictly proper" (Rothenberg 1969, 1975, 1978). In just intonation, tritriadic scales are either strictly proper or improper.*

	TRIAD	MED.	CTS	TETRACHORD	PROPRIETY
1A.	4:5:6	5/4	386	16/15 · 9/8 · 10/9	SP
1B.	10:12:15	6/5	316	10/9 · 9/8 · 16/15	SP
2A.	6:7:9	7/6	267	8/7 · 9/8 · 28/27	I
2B.	14:18:21	9/7	435	28/27 · 9/8 · 8/7	I
3A.	18:22:27	11/9	347	12/11 · 9/8 · 88/81	SP
3B.	22:27:33	27/22	355	88/81 · 9/8 · 12/11	SP
4A.	26:32:39	16/13	359	13/12 · 9/8 · 128/117	SP
4B.	32:39:48	39/32	342	128/117 · 9/8 · 13/12	SP
5A.	22:28:33	14/11	418	22/21 · 9/8 · 112/99	I
5B.	28:33:42	33/28	284	112/99 · 9/8 · 22/21	I
6A.	10:13:15	13/10	454	40/39 · 9/8 · 52/45	I
6B.	26:30:39	15/13	248	52/45 · 9/8 · 40/39	I
7A.	22:26:33	13/11	289	44/39 · 9/8 · 104/99	I
7B.	26:33:39	33/26	413	104/99 · 9/8 · 44/39	I
8A.	14:17:21	17/14	336	56/51 · 9/8 · 68/63	SP
8B.	34:42:51	21/17	366	68/63 · 9/8 · 56/51	SP
9A.	16:19:24	19/16	298	64/57 · 9/8 · 19/18	I
9B.	38:48:57	24/19	404	19/18 · 9/8 · 64/57	I
10A.	64:81:96	81/64	408	256/243 · 9/8 · 9/8	I
10B.	54:64:81	32/27	294	9/8 · 9/8 · 256/243	I
11A.	26:34:39	17/13	464	52/51 · 9/8 · 136/11	I
11B.	34:39:51	39/34	238	136/117 · 9/8 · 52/51	I
12A.	14:16:21	8/7	231	7/6 · 9/8 · 64/63	I
12B.	16:21:24	21/16	471	64/63 · 9/8 · 7/6	I
13A.	20:23:30	23/20	242	80/69 · 9/8 · 46/45	I
13B.	46:60:69	30/23	460	56/45 · 9/8 · 80/69	I
14A.	18:23:27	23/18	424	24/23 · 9/8 · 92/81	I
14B.	46:54:69	27/23	278	92/81 · 9/8 · 24/23	I
15A.	38:46:57	23/19	331	184/171 · 9/8 · 76/69	SP
15B.	46:57:69	57/46	371	76/69 · 9/8 · 184/171	SP

7-5. Mixed tritriadic scales. The triads are 4:5:6 and 6:7:9. (Poole 1850). Mixed scales may often be decomposed into two tetrachords and a disjunctive tone in more than one way. Farnsworth's scale is a mode of Poole's. It may be construed as a tonic major triad, a dominant seventh chord, or a septimal minor triad (6:7:9) on the supertonic (Farnsworth 1958, 1969).

POOLE'S "DOUBLE DIATONIC" OR "DICHORDAL SCALE"

SUBDOMINANT	4/3 5/3 2/1	*2/d x 2/1*
TONIC	1/1 5/4 3/2	*1/1 m d*
DOMINANT	3/2 7/4 9/8	*d s* d^2

1/1 9/8 5/4 4/3 3/2 5/3 7/4 2/1
9/8 · 10/9 · 16/15 · 9/8 · 10/9 · 21/20 · 8/7

ALTERNATE TETRACHORDAL FORM

1/1 10/9 7/6 4/3 3/2 5/3 16/9 2/1
10/9 · 21/20 · 8/7 · 9/8 · 10/9 · 16/15 · 9/8

FARNSWORTH'S SCALE

SUBDOMINANT	21/16 27/16 2/1	*d·s* d^3 *2/d*
TONIC	1/1 5/4 3/2	*1/1 m d*
DOMINANT	3/2 15/8 9/8 21/16	*d d·m* d^2 *d·s*

1/1 9/8 5/4 21/16 3/2 27/16 15/8 2/1
9/8 · 10/9 · 21/20 · 8/7 · 9/8 · 10/9 · 16/15

TETRACHORDAL FORM

1/1 9/8 5/4 4/3 3/2 5/3 7/4 2/1
9/8 · 10/9 · 16/15 · 9/8 · 10/9 · 21/20 · 8/7

corresponding septimal minor and septimal major scales. The septimal minor or subminor scale sounds rather soft and mysterious, but the septimal major is surprisingly harsh and discordant. Triads 9a and 9b are virtually equally tempered and sound very much like their 12-tone counterparts. The scales based on 10a and 10b are the Pythagorean tunings of the major and minor modes in which the thirds are the brilliant, if somewhat discordant, 81/64 and 32/27.

Triads with *undecimal*, *tridecimal*, and *septendecimal* thirds (numbers 3a–8b of 7-4) are less consonant than those discussed above. However, these triads are still relatively smooth and may be useful in certain contexts. Their tetrachords are also interesting melodically as they approximate certain medieval Islamic and neo-Aristoxenian genera (chapter 4). The tetrachords generated by the even less harmonious triads 24:31:36, 64:75:96, 34:40:51, 30:38:45, and 24:29:36 and their conjugates will be found in the Main Catalog.

Scales with mixed triads

Tritriadic scales may also be constructed from triads with different mediants, provided that *d* remains 3/2. An example where the tonic and subdominant triads are 4:5:6 and the dominant triad is 6:7:9 is shown in 7-5 (Helmholtz [1877] 1954, 474). The tetrachordal structure may be described as 9/8 · 8*m*/9 · 4/3*m* (where *m* is the mediant of the tonic triad) for the lower tetrachord and 2*x*/3 · *s*/*x* · 2/*s* (where *x* and *s* are the sixth and seventh of the scale) for the upper tetrachord. However, as 7-5 indicates, mixed tritriadics may often be divided into two tetrachords and a disjunctive tone is more than one way.

Farnsworth's scale, also shown in 7-5, is a mode of Poole's Double Diatonic (Farnsworth 1969). It may be construed as a major triad on 1/1, a dominant seventh chord on 3/2, and a subminor triad (6:7:9) on 9/8.

In chapter 5, the limits on the propriety of mixed modes are discussed.

Ellis's duodenes

Composers may find the intrinsic harmonic resources of tetrachordal scales rather sparse, even with the addition of one or more historically motivated supplementary tones. Two simple remedies immediately come to mind. One is to enlarge the chain of chordal roots of tritriadic scales to encompass four or more triads. This procedure may tend to hide the tetrachords beneath a mass of chords, but by way of compensation, more tetrachords are created. The process may be seen in 7-6. The parent tritriadic scale contains five tetrachords, all of which are permutations of 16/15 · 9/8 · 10/9 (112 + 204 + 182 cents). The new *pentatriadic* scale contains 42 tetrachords of six different genera.

The second solution is to extend both the *d* and *m* axes to generate structures analogous to A. J. Ellis's *duodenes*, the twelve note "units of modulation" in his theory of just intonation in European tonal harmony (Helmholtz [1877] 1954). The duodene generated from the 4:5:6 triad and some analogs generated by other triads are illustrated in 7-7. These scales likewise consist of large numbers of tetrachords of diverse genera in a harmonic context.

Perrett's harmonizations

Wilfrid Perrett, an English theorist, developed some highly imaginative, if controversial, ideas about Greek music and its early history. In *Some Questions of Musical Theory*, Perrett harmonized a version of the enharmonic tetrachord (21/20 · 64/63 · 5/4) which he attributed to Tartini, but it is more likely that Pachymeres has priority. Perrett used familiar tonic, subdominant, and dominant chord progressions by adding tones, effectively embedding the tetrachord in a larger microchromatic gamut (Perrett 1926, 1928, 1931, 1934). It is this harmonization that Partch quoted in *Genesis of*

7-6. *Pentatriadic scales. A pentatriadic is an expansion of a tritriadic by the addition of the subdominant of the subdominant and the dominant of the dominant. An alternative form has a third dominant in place of the second subdominant and is a mode of the scale above.*

THE 4:5:6 TRIAD AND A DERIVED PENTATRIADIC SCALE

	16/9 10/9 4/3	$2/d^2$ m/d^2 $2/d$
SUBDOMINANT	4/3 5/3 2/1	$2/d$ m/d $2/1$
TONIC	1/1 5/4 3/2	$1/1$ m d
DOMINANT	3/2 15/8 9/8	d $d{\cdot}m$ d^2
	9/8 45/32 27/16	d^2 $m{\cdot}d^2$ d^3

1/1 10/9 9/8 5/4 4/3 45/32 3/2 5/3 27/16 16/9 15/8 2/1
10/9·81/80·10/9·16/15·135/128·16/15·10/9·81/80·256/243·135/128·16/15

TETRACHORDS IN SCALE

RATIOS	CENTS	NUMBER
1. 81/80 · 256/243 · 5/4	22 + 90 + 396	3
2. 256/243 · 135/128 · 6/5	90 + 92 + 316	3
3. 135/128 · 16/15 · 32/27	92 + 112 + 294	8
4. 81/80 · 10/9 · 32/27	22 + 182 + 294	7
5. 16/15 · 9/8 · 10/9	112 + 204 + 182	18
6. 256/243 · 9/8 · 9/8	90 + 204 + 204	3

a Music (Partch [1949] 1974, 171). Perrett placed the tetrachord in the soprano voice and added sufficient extra tones in the lower registers to obtain the desired chord progression. 7-8 simplifies Partch's presentation by leaving out the repeated chords under 16/15, 21/20, and 1/1 that follow the one under 4/3, and by transposing the pitches from 5/3 to 1/1.

Perrett also devised harmonizations for a number of other tetrachords listed by Ptolemy. These harmonizations are shown in 7-9 where they have been transposed to 1/1 and tabulated in a standard format.

Perrett also discovered a harmonization of Archytas's enharmonic, 28/27 · 36/35 · 5/4, a much more plausible and consonant tuning than the 21/20 · 64/63 · 5/4 he chose initially (Perrett 1928, 95). He expressed the solution in the 171-tone equal temperament and later translated it into a

7-7. *Ellis's duodenes. This table is based on Helmholtz [1877] 1954, 457-464. The axes have been reversed from the original in which the chain of 3/2's was vertical. Note the interlocking prime (major) and conjugate (minor) triads. The 4:5:6 duodene contains 54 tetrachords of diverse genera. 10:12:15 is a conjugate duodene which should be compared with the one above of which it is not a "mode." It contains 48 tetrachords of different genera. 6:7:9 is a non-tertian duodene. It contains 62 tetrachords of various genera.*

TRADITIONAL DUODENE BASED ON THE 4:5:6 TRIAD

5/3	5/4	15/8	45/32
4/3	1/1	3/2	9/8
16/15	8/5	6/5	9/5

DUODENE BASED ON THE 10:12:15 TRIAD

8/5	6/5	9/5	27/20
4/3	1/1	3/2	9/8
10/9	5/3	5/4	15/8

DUODENE BASED ON THE 6:7:9 TRIAD

14/9	7/6	7/4	21/16
4/3	1/1	3/2	9/8
8/7	12/7	9/7	27/14

7-8. *Perrett's harmonization of Pachymeres's enharmonic. The numbers under the note ratios represent the harmonic factors or Partch "Identities" of the chords. The uppermost voice contains the tones of the tetrachord. The ratios of each of the chordal components are shown below. Asterisks indicate the roots of harmonic chords, "Otonalities" in Partch's nomenclature. The 28/15 does not occur in the Partch gamut, but a transposed version is available in Partch's system starting on 1/1 = 5/3. The pitches of the tetrachord then become 5/3 7/4 16/9 and 10/9.*

1/1	21/20	16/15	4/3
5	7	8	5
4	6	7	4
3	5	6	3
1	1	1	1
5 = 2/1	7 = 21/20	8 = 16/15	5 = 4/3
4 = 8/5	6 = 9/5	7 = 28/15	4 = 16/15
3 = 6/5	5 = 3/2	6 = 8/5	3 = 8/5
1 = 8/5	1 = 6/5	1 = 16/15	1 = 16/15
8/5 *	6/5 *	16/15 *	16/15 *

17-limit just intonation (Perrett 1934, 158). This harmonization is shown as number 7 of 7-9.

I have devised another harmonization, which is noteworthy in that the movement between the roots of last two chords of the cadence is by a 40/27 rather than a 3/2. This example is shown in 7-10.

These harmonizations are rather simple, with few nonharmonic tones or passing chords. More sophisticated techniques including the use of subharmonic chords would seem appropriate.

More complex treatment is obviously possible in larger microchromatic scales such as Partch's 43-tone gamut. With the help of a computer, 4022 occurrences of tetrachords and 1301 heptatonic scales in which both tetrachords are identical have been found in this scale. Among these are the instances of the *Ptolemaic sequence*, Partch's name for the major mode, and a number of other tetrachords from Ptolemy's catalog. Smaller systems such as Perrett's 19-tone scale have considerable tetrachordal resources; 269 tetrachords and 52 heptatonic tetrachordal scales occur in this gamut.

7-9. Perrett's other tetrachord harmonizations. The names for numbers 3 and 4 are Perrett's; the tetrachord is actually Archytas's diatonic and Ptolemy's tonic diatonic genus rearranged. In ascending form, the tetrachord of numbers 1 and 6 is 28/27 · 15/14 · 6/5, Ptolemy's soft chromatic.

1. INVERTED PTOLEMY'S SOFT CHROMATIC

1/1	6/5	9/7	4/3
5	5	9	7
4	6	7	6
3	4	5	5
1	1	2	1

2. PTOLEMY'S SOFT CHROMATIC

1/1	28/27	10/9	4/3
6	7	5	6
5	6	4	5
4	5	3	4
1	1	1	1

3. PTOLEMY'S "SOFT DIATONIC," REARRANGED

1/1	28/27	7/6	4/3
6	7	7	8
5	6	6	7
4	5	5	6
1	1	1	1

4. PTOLEMY'S "SOFT DIATONIC," REARRANGED, ALTERNATIVE CHORDS

1/1	28/27	7/6	4/3
6	7	5	8
5	6	4	7
4	5	3	6
1	1	1	1

5. ARCHYTAS'S DIATONIC

1/1	28/27	32/27	4/3
6	14	16	16
5	12	12	12
4	9	9	8
2	4	6	5

6. INVERTED PTOLEMY'S SOFT CHROMATIC, ALTERNATIVE CHORDS

1/1	6/5	9/7	4/3
5	5	90	20
4	6	70	15
3	4	63	12
1	1	45	10

7. ARCHYTAS'S ENHARMONIC

1/1	28/27	16/15	4/3
8—16	12	28	6
5—10	10	24	5
3—7	7	17	4
2—4	4	10	

7-10. *Another harmonization of Archytas's enharmonic. The root of the chord under 28/27 is 40/27 a syntonic comma lower than 3/2. The septimal tetrad on 16/15 lacks a major third.*

1/1	28/27	16/15	4/3
5	7	8	5
4	6	7	4
3	5	6	3
1	1	1	1

Many of these tetrachords closely approximate divisions based on higher harmonics or equal temperaments, such as those found in Aristoxenian theory. Because they are composed of secondary or multiple number ratios whose factors are limited to 11, their tones may be harmonized by comparatively simple harmonic or subharmonic chords in a tetradic or hexadic texture.

Wilson's expansions

Perhaps the most innovative technique for harmonizing tetrachords is due to Ervin Wilson (personal communication, 1964). Wilson's technique is based on sequences of chords of increasing intervallic span linked by a common tone. Wilson's have the property that the successive differences between the chordal factors follow a consistent pattern. This pattern is termed the ***unit-proportion*** (UP). It controls both the rate of intervallic expansion and less directly the degree of consonance. For harmonic chords, it may be expressed as a string of signed, positive integers, i.e., the unit-proportion of the major triad 4:5:6:8 is +1 +1 +2. Subharmonic unit-proportions are written with prefixed – signs; the unit-proportion of the chord 8:6:5:4 is –2 –1 –1. Sequences of chords with identical unit-proportions make up an expansion which progresses from a dense, relatively discordant chord through chords of decreasing tension to a stable consonance, usually a triad with the root doubled.

Sequences of such chords may be used in many musical contexts, and somewhat similar chordal sequences have been explored by Fokker (1966, 1975). Wilson's expansions are particularly attractive when applied to tetrachords and tetrachordal scales.

The application of Wilson's technique to tetrachordal scales is best seen by example. Wilson's original examples were harmonizations of the inverted enharmonic genera, 1/1 5/4 9/7 4/3 (Archytas) and 1/1 5/4 13/10 4/3 (Avicenna) approximated in 22- and 31-tone equal temperament. These examples have been translated into just intonation and are shown in 7-11. An optional 7:8:9:11 chord has been added to Wilson's original progression for the inverted Archytas's enharmonic.

Although one may limit the harmonization to a single tetrachord, it is more likely that one will want to harmonize all seven tones of the scale. Several solutions to this rather difficult problem using both harmonic and subharmonic chords with varied unit-proportions and different common tones are given in 7-12. In these examples, either the 4/3 or 3/2 is held constant throughout the progression. A passing chord containing intervals of 13 and 15 is used in number 2 to make the progression smoother. These intervals are conditioned in part by the unit-proportion of the set and in part by the intervals of the tetrachord. The major caveat is to limit the number of chords and extra tones when preservation of the melody of the tetrachord is important.

Except for octave transposition of some of the chordal tones and ocassional passing chords there has not been much study of harmonic elaboration (Wilson, personal communication). This is true of the endogenous and tritriadic approaches as well. The standard techniques, however, would appear to be applicable here as in traditional practice, but only more experimentation will tell.

Although the majority of this chapter has been presented from the viewpoint of just intonation, these scales and their various harmonizations are equally valid in systems of equal temperament which furnish adequate approximations to the important melodic and harmonic intervals.

7-11. *Wilson's expansion technique. The set of ratios are the chordal tones relative to 1/1. (1) is the just intonation version of Wilson's first expansion harmonization with the later addition of an optional 7 8 9 11 chord at the beginning. The original was quantized to 22-tone equal temperament. (2) is the just intonation version of Wilson's second expansion harmonization. The original was quantized to 31-tone equal temperament. In both cases, the added tones are in lighter type. The optional chord is in parentheses.*

1. INVERTED ARCHYTAS ENHARMONIC, HARMONIC CHORDS ON 3/2, UP = +1 +1 +2

1/1				5/4	9/7		4/3	3/2		15/8	27/14	2/1
			(7				8	9	11)			
			(7/6				4/3	3/2	11/6)			
		6				7		8		10		
		9/8				21/16		3/2		15/8		
	5				6			7			9	
	15/14				9/7			3/2			27/14	
4				5				6				8
1/1				5/4				3/2				2/1

2. INVERTED AVICENNA'S ENHARMONIC, HARMONIC CHORDS ON 3/2, UP = +3 +3 +6

1/1			5/4		13/10		4/3	3/2	15/8	39/20	2/1
		18				21		24	30		
		9/8				21/16		3/2	15/8		
	14			17				20		26	
	21/20			51/40				3/2		39/20	
12			15					18			24
1/1			5/4					3/2			2/1

7-12. Trial expansion harmonizations. The successive differences or unit proportions are positive in harmonic chords, negative in subharmonic. The non-scalar added tones are in lighter type. Passing notes are in parentheses.

1. DIDYMOS'S CHROMATIC, SUBHARMONIC CHORDS ON 4/3,
UP = −5 −3 −2

1/1 16/15 10/9 4/3 3/2 8/5 5/3 2/1

30 25 22 20
10/9 4/3 50/33 5/3

25 20 17 15
16/15 4/3 80/51 16/9

20 15 12 10
1/1 4/3 5/3 2/1

2. HARMONIC CHORDS, 3/2 COMMON, PASSING NOTES INSERTED,
UP = +1 +2 +3

1/1 7/6 5/4 4/3 3/2 7/4 15/8 2/1

15 16 18 21
5/4 4/3 3/2 7/4

(12) (13) 15 (18)
(6/5) (13/10) 3/2 (9/5)

9 10 12 15
9/8 5/4 3/2 15/8

6 7 9 12
1/1 7/6 3/2 2/1

3. ARCHYTAS'S ENHARMONIC, SUBHARMONIC CHORDS ON 4/3,
UP = +2 −1 −1

1/1 28/27 16/15 4/3 3/2 14/9 8/5 2/1

11 9 8 7
12/11 4/3 3/2 12/7

10 8 7 6
16/15 4/3 32/21 16/9

9 7 6 5
28/27 4/3 14/9 28/15

8 6 5 4
1/1 4/3 8/5 2/1

4. INVERTED DIDYMOS'S CHROMATIC, HARMONIC CHORDS ON 3/2,
UP = +2 +3 +5

1/1 6/5 5/4 4/3 3/2 9/5 15/8 2/1

20 22 25 30
6/5 33/25 3/2 9/5

15 17 20 25
9/8 51/40 3/2 15/8

10 12 15 20
1/1 6/5 3/2 2/1

5. ARCHYTAS'S ENHARMONIC, 4/3 COMMON, HARMONIC CHORDS,
UP = +2 +2 +2

1/1 28/27 16/15 4/3 3/2 14/9 8/5 2/1

14 16 18 20
7/6 4/3 3/2 5/3

10 12 14 16
10/9 4/3 14/9 16/9

8 10 12 14
16/15 4/3 8/5 28/15

7 9 11 13
28/27 4/3 44/27 52/27

6 8 10 12
1/1 4/3 5/3 2/1

6. INVERTED ARCHYTAS'S ENHARMONIC, SUBHARMONIC CHORDS ON
3/2, UP = −2 −2 −2

1/1 5/4 9/7 4/3 3/2 15/8 27/14 2/1

20 18 16 14
6/5 4/3 3/2 12/7

16 14 12 10
15/14 5/4 3/2 15/8

14 12 10 8
27/26 27/22 3/2 27/14

13 11 9 7
9/8 9/7 3/2 9/5

12 10 8 6
1/1 6/5 3/2 2/1