Yet Another 72-Noter

Introduction

As I said in the letter that was published in the 1987 Winter issue, I am delighted to see Computer Music Journal taking such an interest in microtones.[1] It was, no doubt, inevitable, now that we have personal computers and systems that can allow us economically to compare tunings arid temperaments and finer divisions of the octave to (almost) our hearts' content.

I have spent most of my microtonal career on the edges of the computer world. The first piece I wrote in what I guess I can call my mature technique (*From an Oboe Quartet* [1971]) was tried out, little by little as it was composed, on the PDP-6 (10) system at M.I.T.'s Artificial Intelligence Laboratory, using Stephen Smoliar's EUTERPE, which he had extended specifically to accommodate my music. In 1985, I convinced Larry Johnson to develop a program for his Commodore 64 that allowed me to provide a computer realization of the vocal pieces mentioned later in this article, to help the singers learn how to associate the notes with the notation. At present, I am working with David Rayna on a 72-note-per-octave keyboard. It is more conventional in arrangement than Joseph Maneri's at New England Conservatory, and runs under computer management. I am at work on a piece that will have computer-generated sound as one of its elements. But being only secondarily a programmer or technician, I can have little to say about computer music per se.

I can bring to a discussion of the matters of intonation, consonance, tuning, and composition with microtones—that filled so much of the microtonality issue of *Computer Music Journal*—something from my practical experience that can usefully complement more theoretical and speculative articles.

For upward of two decades now—since 1964, when I wrote my String Octet—I have written mostly a music requiring a 72-note division of the octave. These can be combined as needed to describe the complete 72-note gamut, as in the sample given in Figure 1. The music uses an 18-note subset of the 72 notes in the same way that tonal music uses the 7-note diatonic subset of the 12. At any moment, there is in effect a transposition of that subset that defines a unique tonal region in exactly the same way transpositions of the diatonic scale do.

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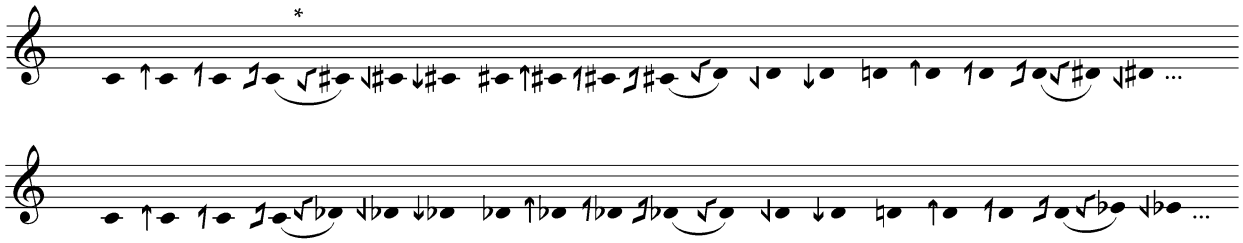
[1] This article first appeared in *Computer Music Journal*, Vol. 12, No. 4, Winter 1988.

The microtone symbols I've used in my 72-note music since 1970.

	Inflection down a 1/12 tone		Inflection up a 1/12 tone
	Inflection down a 1/6 tone		Inflection up a 1/6 tone
	Inflection down a 1/4 tone		Inflection up a 1/4 tone
	Inflection down a 1/2 tone		Inflection up a 1/2 tone

Fig. 1. Initial segment of the complete 72-note gamut



Background

It would not come amiss for me to describe something of my progress toward this. After all, use of more than 12 notes in the octave is still problematical—downright heretical, in some people's minds—so one does well to show how one's acceptance of it was a matter of internal necessity, not visionary utopianism or mere chic.

When I went to college (Birmingham-Southern) in 1944, I found there a Carnegie record set, and in that set I found music by Julian Carrillo (*Preludio a Cristobal Colon*). I found the Carrillo obviously lunatic—very modern, but not an example to follow.

While I was in college, I tried to make a setting of T. S. Eliot's *Death by Water*, but gave it up when I found that I was going to need a falling, stepwise succession of some five or six pitches within, a minor third. I would not have known how to notate them, even if I had thought them respectable.

During that time, I studied under the man who has probably had the most important effect on me of any of my teachers—Hugh Thomas. In the choruses he conducted, he made us exaggerate the inflection of leading tones, the subdominant when part of the dominant seventh, and chromatic tendency tones. A few years of this and, even if you know nothing about the history of tuning and the effects of Pythagorean or Just Intonation, you are liable to find it hard ever again to believe (no

matter how much the keyboard instruments may try to convince you it is so) that there is, for example, one thing which is G-sharp, one frequency that defines it for ever and ever, Amen.

So, when later on I heard Harry Partch's music, I was a bit more ready for it. The idea of harmonizing the speaking voice was interesting and attractive—until I actually heard the *Oedipus* and found that what he called "speaking" was as artificial as singing and a lot less attractive. The music was pretty and the harmonic sound interesting enough to make me think it might be fun to write for those instruments. But neither was pretty or interesting enough to make me willing to submit to Partch's theory and idea of the proper style.

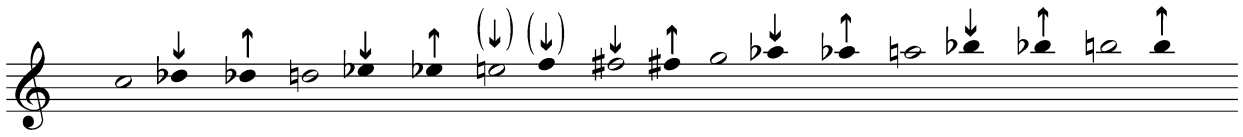
So I kept on being "respectable," all the while learning accidentally more and more about other tunings and temperaments, for example Huyghens' 31-note system, so popular in the Netherlands; the actual differences between harmonics (the 7th and 11th, say) and their tempered equivalents (1/6-tone and 1/4-tone downward inflected minor seventh and augmented fourth, respectively). And I was having trouble composing nondiatonic music away from the piano. When I imagined "half-steps" and wrote them down as tempered ones and then went to try them out, the music did not arrive where it was supposed to.

Finally, in 1960, I could no longer avoid joining the lunatic fringe (which is what, during all my youth, the world of the microtone enthusiasts had looked like to me, and still looks like to most musicians, I gather, even today). It had become obvious that I was going to use the little intervals or none. And, since none of the music I had ever come upon that used them seemed to provide any model I would want to emulate, I had to go by instinct, transcribing as best I could what my ear seemed to be demanding.

Working Out the Vocabulary

I tried, poor brainwashed thing, quarter-tones at first (in, for example, my *Sonate Concertante* [1990], or my *Third Quartet* [1962]—see the bibliography at the end of this article for the publishers and recordings of pieces discussed here). Despite the fact that Penderecki and a few others were making them stylish and therefore at least semirespectable (which had by now long ceased to concern me), I found they would not do. They made compositional thinking easier for me, but not fully so, and performers found them all but impossible to play accurately. A 24-note gamut seems to run counter to Western (all human, I suspect) acoustical instincts.

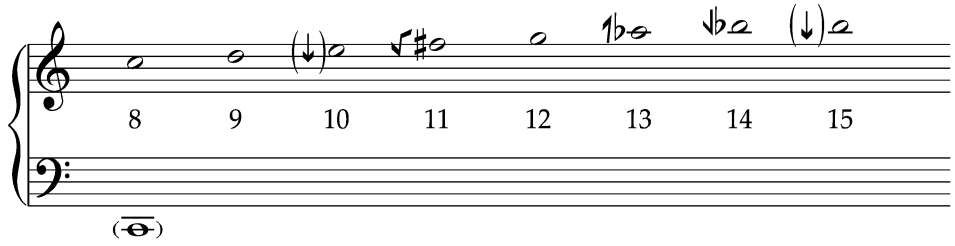
Fig. 2. My initial intuition of the 72-note scale. (The arrows here merely indicate that the note is raised or lowered, by some as-yet-undefined amount, above or below the tempered one. I had not yet established what my notation would be.)



With a little more careful attention, I realized that I was hearing in terms of a scale of the form given in Fig. 2, a dense collection of "chromatic" notes organized around the diatonic scale as a sort of armature.

It was clear to me that in this scale the six steps within each of the major thirds, C-E and G-B, were essentially equal, as were the four within the E-G minor third. It was further apparent to me that the scale determined a tonal region (what I will often call a "key" even though that word ought perhaps be reserved for only the keys of diatonic tonal music). That is to say, the same succession of intervals could begin on another pitch and define a new key, a new tonal region, with a different fundamental, just as transpositions of the diatonic scale do and of the 12-note scale do not. This seemed to imply a structured, asymmetrical set, founded on harmonic relations, like the diatonic scale, not a structureless, symmetrical one like the chromatic.

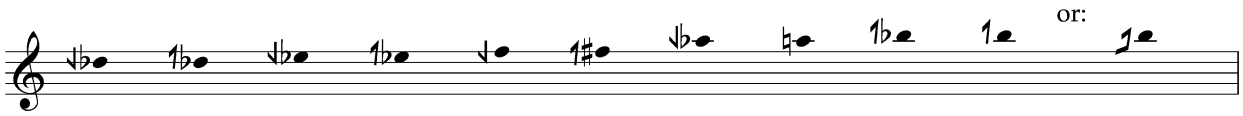
Fig. 3. Eighth through 15th harmonics of C, written in my current notation.



What little acquaintance I had with the facts of acoustic life made me recognize the 8th through the 15th harmonics in there (Fig. 3). Actually, the 13th is a bit higher than that, but not so far that the 1/6-tone-high minor sixth cannot substitute for it, if one wants to be working in equal temperament. The 7th and 11th are quite nicely in tune in equal temperament, and my instinct is that they are the more important intervals: developing a system that keeps the 13th snugly in tune is the next era's concern, not mine.

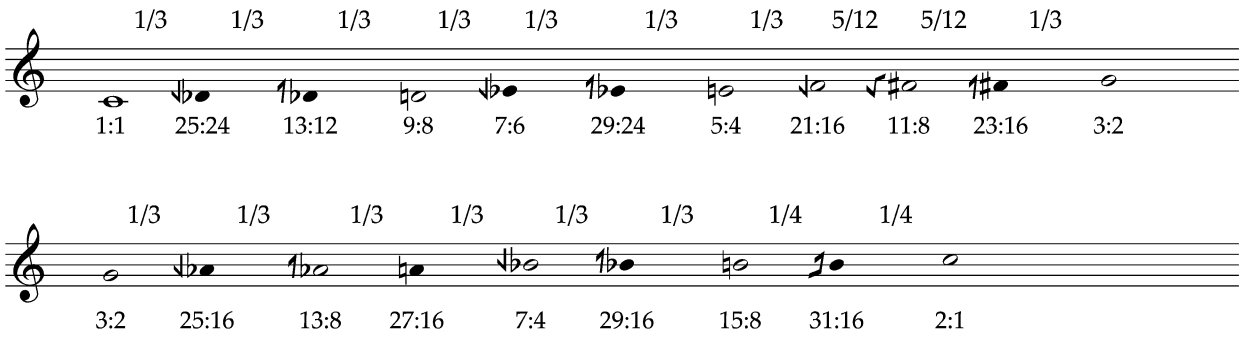
The concept of consonance—what intervals one may end and rest on—in Western music has twice lurched up through the harmonic series: first through the intervals up to the third harmonic (and their inversions), next, through those between the third and sixth. I have noticed in twentieth-century music a strong, if sometimes uncertain, tendency to do the same with the intervals between the 6th and 12th. So I found the apparent congruence of what I was hearing with those elements of the harmonic series reassuring. What I was doing could be put into a not unreasonable (possibly even true) long-term historical perspective. Recognizing this, I thought it seemed sensible—and it did prove comfortable and fruitful—to fill in the rest as Fig. 4.

Fig. 4. The "chromatics" of my scale (in the C transposition).



This makes a full 18-note scale made up of a succession of six 1/3-tones, two 5/12-tones, seven 1/3- tones, and two 1/4-tones. These are shown below and between the notes in Fig. 5.

Fig. 5. My scale, in the C transposition, showing fractions of tones between pitches (above the staff) and harmonic ratios with the fundamental (below the staff).



I identified them with what seemed to me the most appropriate, relatively simple, harmonic ratios (given above the notes in Fig. 6/5), even though I would continue to write them as if equal-tempered, the way we did for tonal music.

I was in those days still thinking in received terms of keyboards and tempered approximations of harmonic ratios. But the life of microtones enlarges the mind. I am still prepared for my music to be played in equal temperament. I am even reconciled to

its being played out of tune (Why should I expect more than Mozart?). But I now think in terms of Just ratios, and, where instruments of fixed pitched are not involved, I really expect the older practice of tuning the current key in something like Just, but adjusting the relations between keys to something like equal temperament in order to avoid going off the instruments. Modulation on these terms is something done every day by good singers and orchestral instrumentalists; now that we have the digital computer to expeditiously provide the Just frequencies for each keynote's scale, it will, I expect, become possible on a manageable keyboard.

FIGURE 6 – Table from Chalmers Book

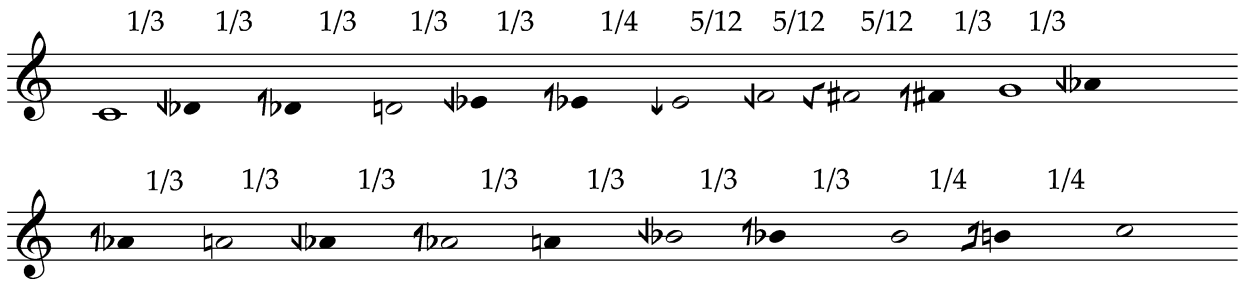
Fig. 6

Scale	*Equal-tempered*	*Just* —*in C*— *in cents*	*in ratios*	*in cents* equal-just*	*Just*	*Difference*
C	1200.00	2/1			1200.00	0
	1150.00		31/16		1145.00	+4
(1b	1133.33		31/16		1145.00	−11.66
B	1083.33	15/8			1088.27	−4.94
1b	1033.33		29/16		1031.00	+2.33
B	966.66	9/4			968.83	−2.17
a	900.00		27/16		905.86	−5.86
A	833.33	13/8			841.53	−8.2
	766.66		25/16		772.63	−5.97
G	700.00	3/2			702.	−2
	633.33		23/16		628.27	+5.06
	550.00	11/8			551.32	−1.31
F	466.66		21/16		470.83	−4.17
E	383.33	5/4			386.31	−2.98
	333.33		29/24		328.39	+4.94
	266.66		7/6		266.83	−.17
D	200.00	9/8			203.91	−3.91
	133.33		13/12		138.5	−5.17
	66.66		25/24		70.67	−4.01
C	0.00	1/1			0.00	0.

* Taken from John Chalmers' *Just Intonation Tables* (1966) and *Conversion Tables* (1970).

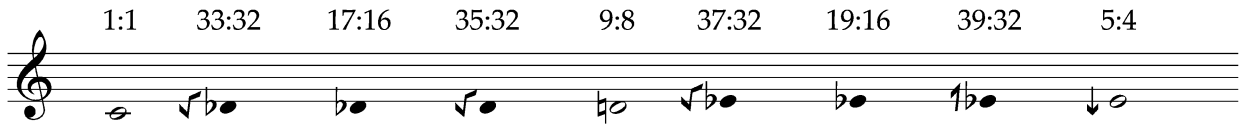
The table in Fig. 6 gives a comparison of the scale as tuned in equal and Just intervals. It soon seemed a good idea generally (but not in every case) to lower the 7th and 17th pitches by 1/12-tone, the nearer to approximate (a little too low) the natural 5th and 15th harmonics, producing a scale containing the stepwise intervals shown in Fig. 7.

Fig. 7. The form of the scale using the lowered 7th and 17th degrees.



Also, the harmonic implication of the first seven pitches make it useful sometimes to have an eighth degree that is a perfect fourth above the tonic, just as other situations can make it desirable to have a progression of 1/4-tones (Fig. 8) instead of the first five notes of the scale as given in Fig. 5.

Fig. 8. Alternate initial segment of my scale (C transposition).



Why 72?

In order to be able to exactly transpose an (ostensibly) equal-tempered scale made up of a mixture of 1/3-, 1/4-, and 5/12-tones to begin on any member of itself and retain the exact same succession of intervals, it was necessary to have a division of the whole tone equal to the least common denominator of the fractions, namely 12. This meant a 72-note octave, just as it had earlier been necessary to have a division of the whole tone using the least common denominator of 1 and 1/2, that is, the chromatic 12-note octave, in order to transpose, the collection of (ostensibly) equal-tempered whole and 1/2-tones that is the diatonic scale to begin on any member of itself and retain the proper succession of its intervals.

This made possible modulation to such keys as that of the 11:8 augmented fourth of C (Fig. 9) or the 13: 8 semimajor sixth (Fig. 10).

Fig. 9. Scale of (INSERT MICRO 3) F sharp, the 11:8 augmented fourth of C. (The fractions indicate fractions of tones, not harmonic ratios).

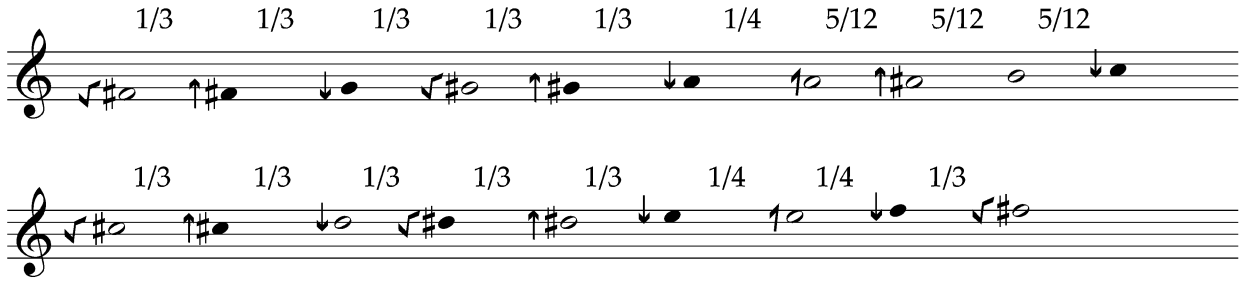
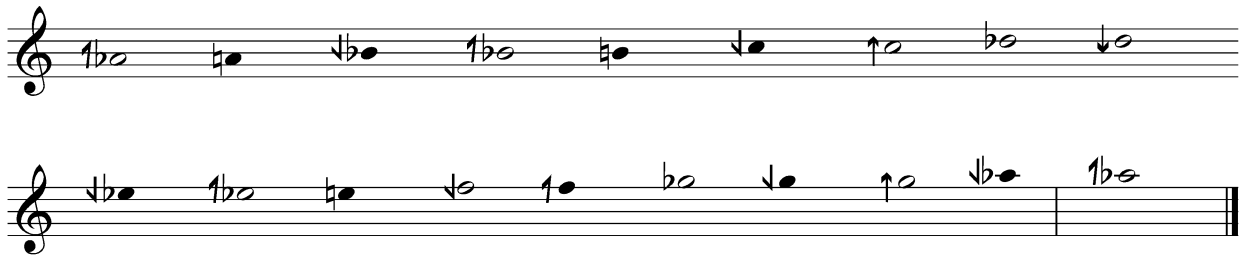


Fig. 10. Scale of (INSERT MICRO 3) A flat, the (approximately) 13:8 semimajor sixth of C.



Notation

When I realized that this was the way I was going to be composing for as long as I could foresee, it behooved me to decide how to notate those pitches. The only ready-made 1/12-tone notation I had ever seen was Haba's, which used these symbols:

HABA's SYMBOLS

If there were symbols for larger intervals down to the whole tone, I didn't—and still don't—want to know them. I knew that if confronted with such a set of symbols, I would give up and go back to modal monody. The graphic distinctions seemed too minute for practical purposes, the symbols too hard to distinguish in either rapid reading or writing, the whole thing too much to ask performers to learn in order to play what was certainly going to be a merely marginal element of their careers for a long while to come. So I worked up my own notation, which used fewer symbols in combination with the familiar accidentals. I have since come on other systems, none of which seems to escape those same drawbacks, so I have kept to my own.

First came the quarter-tone. Since its increment is the square root of the half-step's, and since the square-root radical (square root sign) has a hook that points downward, it seemed inevitable that the two should be associated and the radical be used to indicate inflection by 1/4-tone downward. For upward inflection, merely to turn it upside down did not work very well. It was, for one thing, ungainly to write. So I decided that (symbol) would do. These could be used alone or in combination with the sharp or flat, obviating the need of a 3/4-tone symbol.

Then came the sixth-tone inflection. It, being smaller than the quarter-tone, seemed to want a smaller sign, so it became for downward inflection, for upward. It, too, would be used alone or in combinations like and. These signs could presumably be used in even more complex combinations, like or, but I wanted to avoid any but the most obvious and transparent.

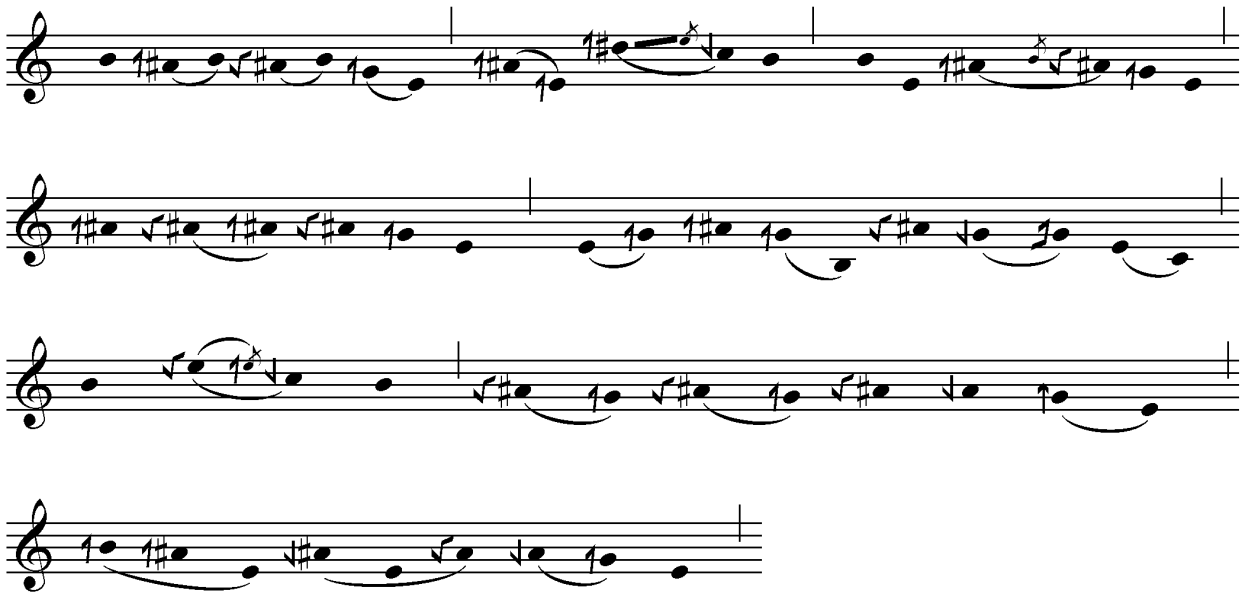
Finally, there was the twelfth-tone inflection. The quarter-tone and sixth-tone symbols related so well, and the twelfth-tone seemed at the time so likely to be much less used, that I decided to leave what I had established as it was and use the full arrow, and, for the twelfth-tone, even though logic would have expected something smaller than the sixth-tone's hook. Performers seem to have had no particular difficulty with all this—much less, I am sure, than they would have had with a notation like Haba's.

Uses and Techniques

Local Relations

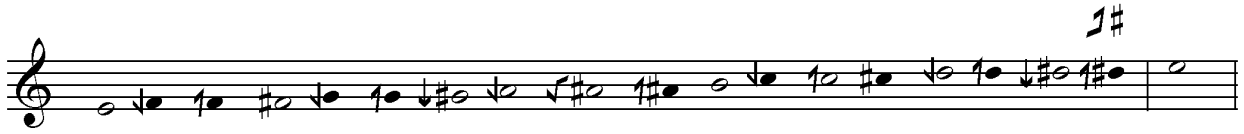
As an instance of the use of this tuning not from my music, let me give Louis Armstrong's performance of *St James Infirmary*. The notes (I ignore the rhythm, which is irrelevant to my point, in any case) of the first vocal chorus are given in Fig. 11.

Fig. 11. The pitches sung by Louis Armstrong in the first vocal chorus of *St James Infirmary*



To my surprise, some people seem to have been taught that the "blue" third is always the small, 7: 6 minor third; some, that it is always the larger, semimajor, one. Armstrong's use of both the small (the sixth-tone low G) and the large (the sixth-tone high G) will come as a surprise to them. These are, however, the pitches, those of the recording made circa 1928 with Earl Hines, reproduced on Columbia 853, checked against a monochord with an equal-tempered scale chart marked in l0-cent increments. An acoustical justification for the use of the semimajor third is not obvious, but one is suggested by the fact that it lies in the same relation to the dominant as the semimajor 13: 8 submediant does to the tonic. With five exceptions (marked in Fig. 11 by asterisks above the notes), which I take to arise from exaggeration for emotional effect, but may have some other explanation altogether (if indeed they need any), all these pitches lie in my scale, in its transposition to E (Fig. 12).

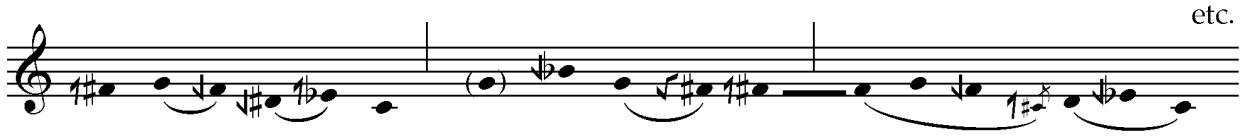
Fig. 12 The E transposition of the scale.



A similar tuning is to be found in quite a lot of jazz and folk performance, and it seems to me no wonder. I should think it inevitable that the 11th and 13th harmonics should inhere in our inborn acoustical machinery (depending to such an extent as it does on

apparently "hardwired" perception of harmonic relations in its interpretation of aural events) in a way that a learned 4:3 subdominant and a 27:16 submediant (or even a 5:3, imported from a different fundamental's series) would not.
My *Sextet* (1983) is based on Armstrong's performance. Figure 13 gives the notes of the first explicit statement of it in the *Sextet*. I retained his emphasis on the 11:8 and 23:16 pitches (the two augmented fourths above the tonic) and semimajor third with its appoggiatura of the small, 7:6, minor third.

Fig. 13 Notes of the first explicit use of *St James Infirmary* in *Sextet* (1983).



Melodically, I use the scale the same way in all my music—though not always so triadically. It seems that those pitches of a region that duplicate the harmonics 8 through 15 (the ones I write in open-note heads when writing out the scale I are more stable, more "consonant," melodically or in contrapuntal combination, than are the ones that represent the higher harmonics (the ones I write in black-note heads), and are the ones that more remote melodic notes or more complex harmonic combinations want to "resolve" to. In the scale of F, for instance, such an interval as (INSERT MUSICAL EXAMPLE) is a less convenient ratio (75:46) than the 16:11 of (INSERT MUSICAL EXAMPLE) and the mind will accept the latter as a resolution of the former, as I hear happening at the opening of my fourth string quartet (Fig. 14).

For another example, the opening notes of my *String Quartet #2* [1962], given in précis in Fig. 15, invoke, by means of the 11:4 of the SYMBOL E: Bb, a fundamental of Bb, then present an accumulation of progressively more complex harmonic ratios. These would have been but a succession of two simple appoggiatura resolutions without the addition of the second pedal on SYMBOL Eb. They resolve it all to the lower energy state of the 8:5 sixth on A, effecting thereby a progression equivalent to the traditional one of tonic to dominant. It is worth pointing out here that, just as the tritone between the fourth and seventh degrees of the diatonic scale could be said uniquely to define the key, the 11:8 augmented fourth in this scale can be said similarly to define the tonal region, particularly if taken in combination with its adjacent pitches, all of them that noticeable 5/12- tone apart.

Fig. 14. String Quartet #4, mm. 1-3.

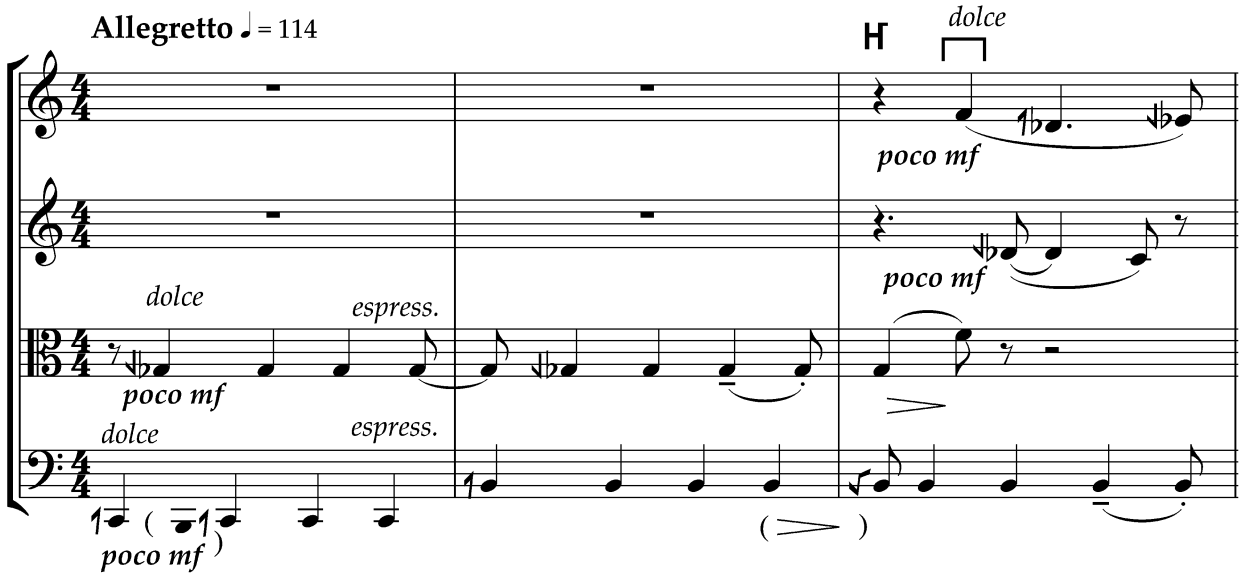
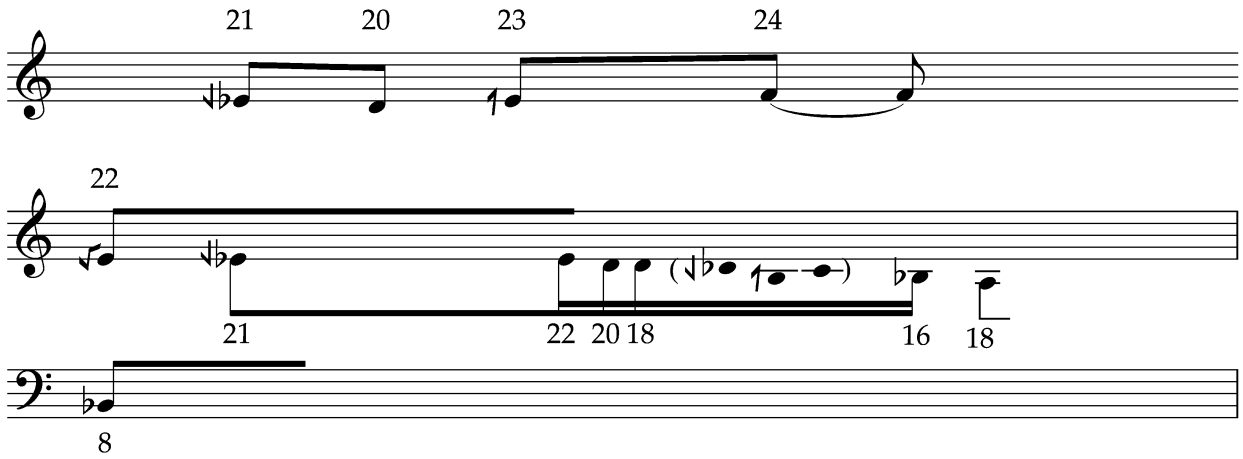


Fig. 15. Précis of the first twelve measures of *String Quartet #2 (1962)* (1974)

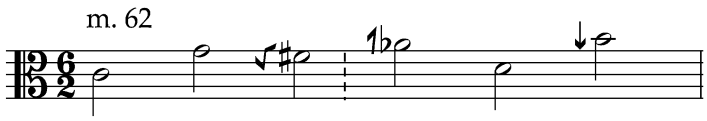


Large Scale Relations

The harmonic relations, which in the scale imply a governing local fundamental, can be used to govern larger formal relationships—such as causing the relations of the fundamentals of the various scales used in the course of a section or whole piece to imply the paramount tonality of that section or place. For example, my *—and, as I was saying,…* [1979] modulates through the regions (scales) of C (mm. 1-9), G (mm. 10-12), r F# (mm. 13-41), 1 Ab (mm. 42-50), D (mm. 51-57), .. B (mm. 55-57), and C (mm. 58-59);

that is, harmonics 1, 3, 11, 12, 9, 15, and 1 of C. to emphasize the point, it then enters onto a coda that uses as its melodic material that same sequence of pitches (Fig. 16)

Fig. 16. First measure of the coda of *– as I was saying*.



Treatment of this line takes the piece momentarily into a tonally indeterminate region, from which it returns, at measure 66, to C, where it reiterates the sequence several times, gradually placing the pitches in their proper overtone spacing, emphasizing thereby the tonic that has been implied by the relation of the fundamentals of the scales traversed in the course of the piece, and leaving the satisfaction of resolving the 15th harmonic to the mind of the listener.

Fig. 17. Opening tune of *Two for One*.

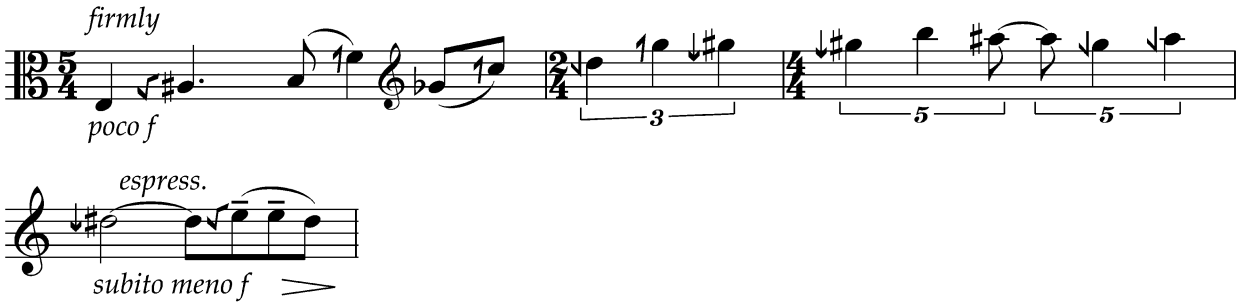


Fig. 18a. Order of harmonics in the tune of *Two for One*.

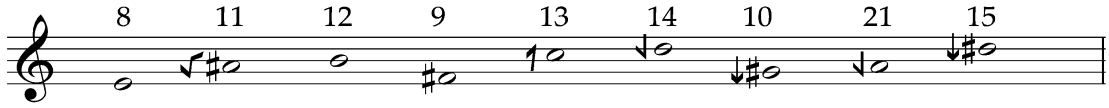
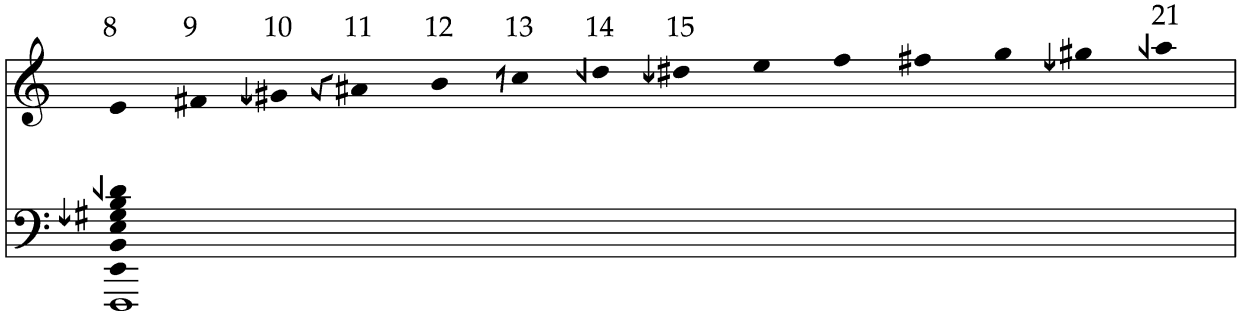
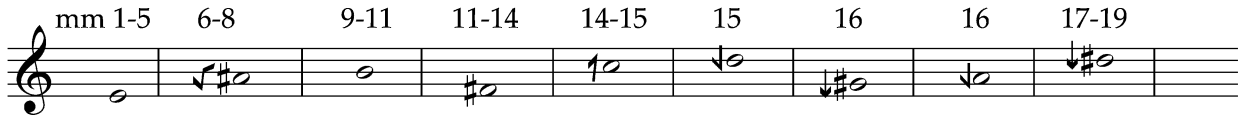


Fig. 18b. Harmonic Series of E flat.



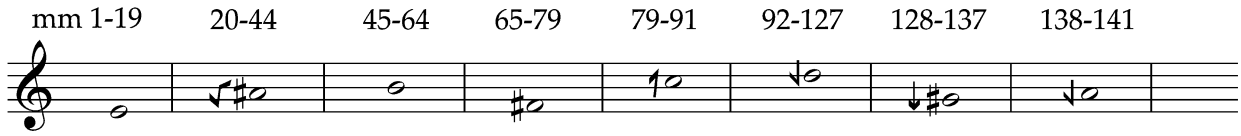
This bespeaks an identity of harmonic, tonal, and melodic disposition, a method I have used since my earliest pieces in the technique. For instance, in *Two for One* (1980), the opening tune (Fig. 17) contains harmonics 8-15 and 21 of E in the order 8, 11, 12, 9, 13, 14, 10, 21 (displaced an octave down) and 15 (Fig. 18). The piece thrice modulates through a succession of regions, the fundamentals of which are in essentially that same sequence: measures 1-19 move quickly through it, touching on some of the regions only briefly (a half-measure each) and in only the most lightly allusive way (Fig. 19).

Fig. 19. Regions of *Two for One*, mm. 1-19.



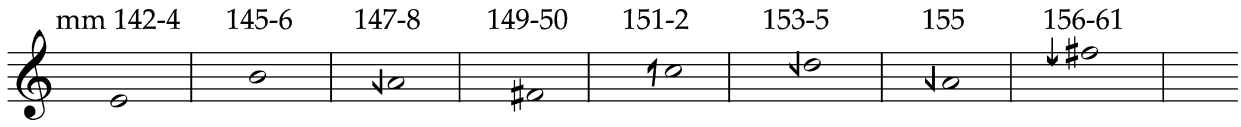
These can be heard to define an overall tonality of E for the section, and mm. 1-141 therefore to modulate through all but the last of the sequence (Fig. 20)

Fig. 20. Regions of *Two for One*, mm. 1-141.



Mm. 142-161 do so again, with certain changes to help effect what in tonal technique would have been obtained by a descent into the sub dominant (Fig. 21).

Fig. 21. Regions of *Two for One*, mm. 142-161.



As with mm. 1-19 locally, this triple succession of keys can be heard to define an overall tonality of E for the entire piece, all the fundamentals of the various regions lying in the harmonic series of E as they do.

The final return to the opening material sticks to the overall fundamental region, E, with passing reference to that of SYMBOL G, omitted from the previous modulations, and

ends with an extended reiteration of the characteristic E SYMBOL interval, the 11:8 of E.

The Harmonic Series as Theme

The harmonic relations implicit in the scale made irresistible the idea of taking the harmonic series itself as a kind of *grundgestalt* and submitting it to various manipulations. The result was my *Quartet* [1982] for flute and strings. The overtone series of E flat can be written (with varying, but not fatal, degrees of inexactness) as shown in Fig. 22. The opening of the quartet is shown in Fig. 23.

Fig. 22. Overtone series of E flat.

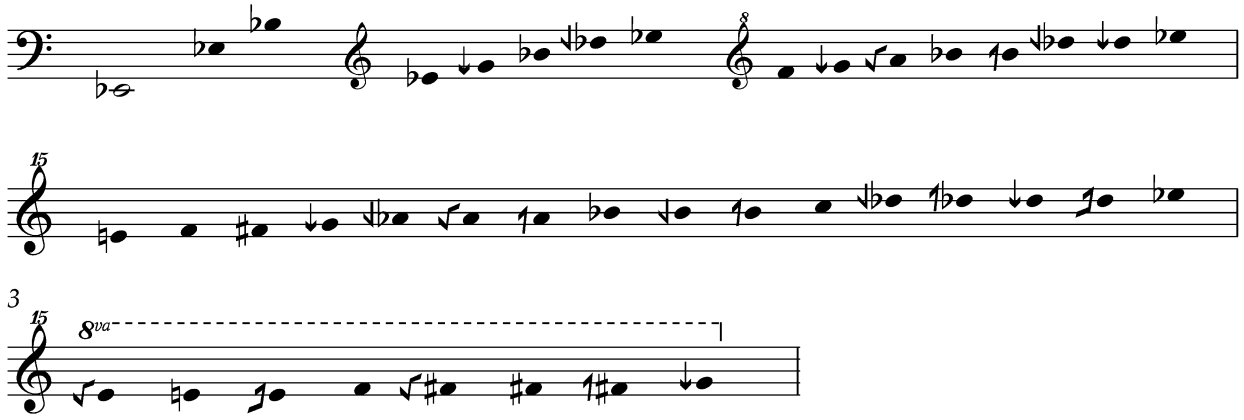
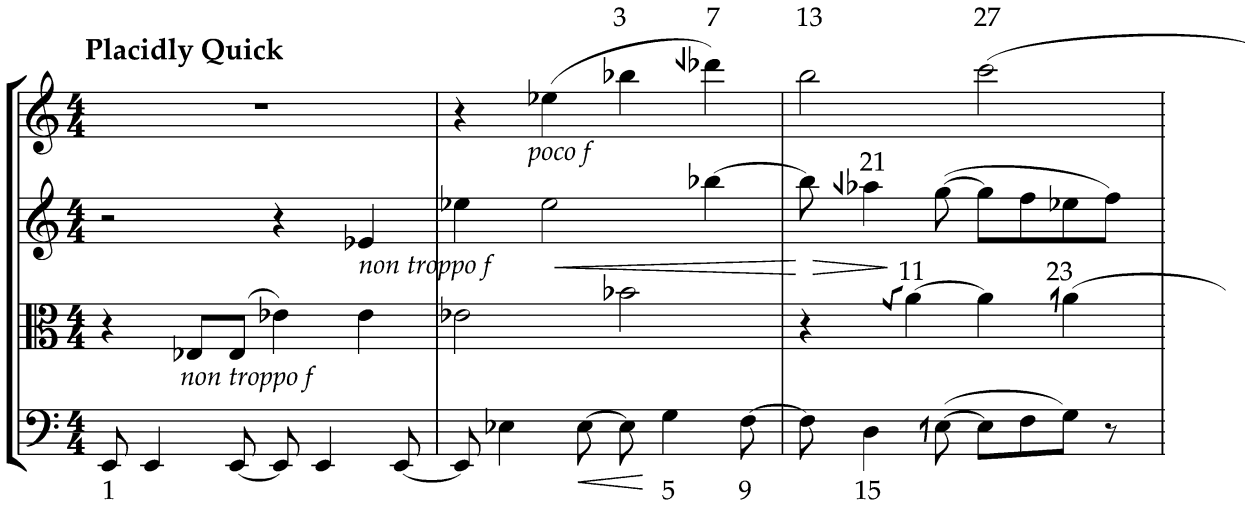
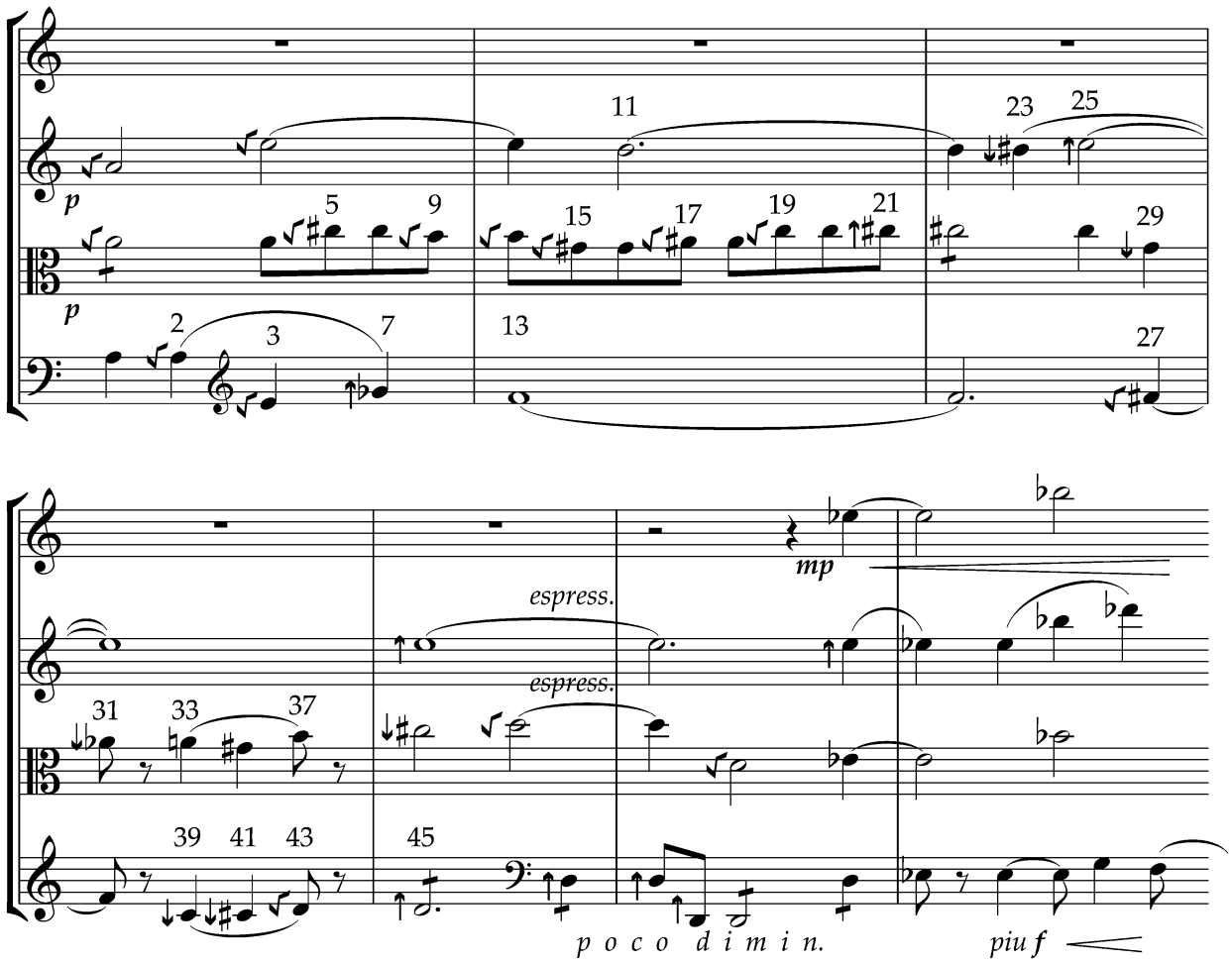


Fig. 23. *Quartet* (1982), mm. 1-3.



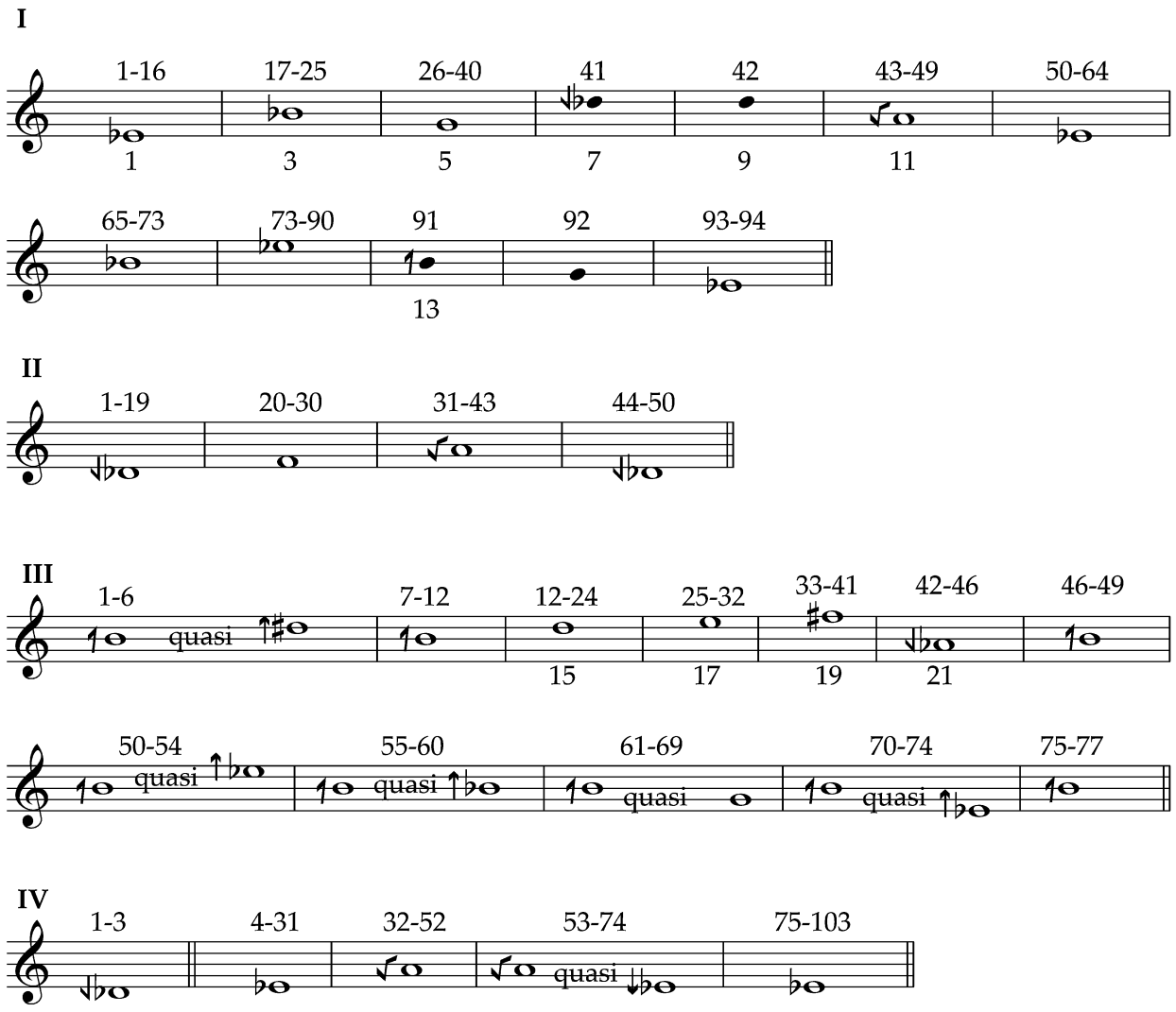
Except for a couple of retardations and one inharmonic passing tone, each new note appears in harmonic series order.
The first movement's development section, a transition back to a recapitulation of the first half in classic manner (it is a humorous and therefore mannerist piece), is a traversal of the harmonic series of SYMBOL A, the 11:8 of Eb (Fig. 24).

Fig. 24. Development section (mm. 44-50) of *Quartet* (1982).



The key sequence of the whole piece is given in Fig. 25. Notice how each new key is introduced in harmonic series order and the last movement emphasizes the 11:8 relation.

Fig. 25. The key sequence of *Quartet* (1982).



Tonal Transposition and "Relative" Keys

The reader will have long since noticed the asymmetry of my scale and its similarity in that respect to the diatonic. That asymmetry has restored to me the old-fashioned pleasure of tonal transposition, largely lost to us nowadays, when much music treats the 12-note gamut as a featureless collection of pitches related only to one another, and all alteration is perforce development. Thus it allows me something analogous to the old notion of relative keys, modes drawn from the same set of pitches but with different centers of gravity, useful for establishing relations that would have been obscured by modulation into an altogether different scale, or for refreshing a tonality by drawing it

from a different, but related, one. Such goings on are shown in Fig. 25 at III (1-6), where the seeming key center is SYMBOL D, but the pitches are those of the scale of SYMBOL B. Similarly, in III (50-77), the apparent key centers again suggest an overall key of Eb, but the actual pitches are again those of B. In IV (53-74), using the notes of A as if their fundamental were Eb produces the effect of a "parallel minor." That the seeming tonic is 1/12-tone lower than the paramount Eb makes the effect the stronger and the section in Eb that follows the brighter.

Harmony

I have not gone far into vertical matters, since my practice there is still so various as to preclude codification, at least by me. It should be sufficient to say that I tend, as I said earlier, to hear those scale degrees that represent the 8th through 15th harmonics as stable, and able to form stable harmonies in combinations of up to eight notes (I generally do not go so far). The combinations can be triadic, quartal, secundal, depending on the requirements of the piece, or all sorts mixed, if need be.
Something else has me interested these days.
This might be called the scale's inherent "harmonicity," a matter of putative resultant tones—summation or differences—acting as melodic, harmonic, or perhaps even structural determinants. I first realized that my ear was attending to them while I was writing my first string quartet in 1958. Since then, I have more and more often found that an awareness of their possibility in some ideal performance could be very useful. This is facilitated by dealing with the ideal situation—taking the notes of the scale as members of a set of Just ratios, in terms of their harmonic numbers rather than their actual frequencies. Thus measures 33-51 of "The Chief's Speech," in my *The Conversions* (1985) (Fig. 26) can be shown to have to do with difference tones, as is shown in Fig. 27. Measures 51-63 (Fig. 28) have to do with summation tones)Fig. 29). And measures 64-75 (Fig. 30) have to do with both (Fig. 31). Lest it be thought I did not notice departures from rigorous adherence to the ideal resultants, let me hasten to say that those departures are there for musical reasons—in my thinking, for dissonance.

It was fascinating to observe in rehearsal and performance of the piece how much this aided good intonation. The resultant formations apparently represent a sort of "lowest energy state", a homeostatic complex requiring less effort of larynx and mind than would any collection of nearby but inharmonic pitches.

This suggests a criterion for dissonance: in the tonal key of C, the major third C-E is consonant even when played out of tune and beating, while the major seventh is dissonant even when in perfect 15:8 tune and presumably beatless. The criterion is the function of the interval in that ideal complex of relations called the key, not its

momentary (and too often accidental) prettiness or lack thereof. I wonder if that does not have to do somehow with the ideal resultants implied by the harmony and nearby pitches.

I am not prepared to go any distance into the matter here, but I can say that I have observed things happen in my music that would make it seem so. An instance occurs several times in my *Quintet* (1987) for clarinet and strings. There are moments in the slow second movement where the second violin holds through most of a phrase, a pedal, the 17th harmonic of the current tonality, which is the resultant tone of the notes of the clarinet and cello. At the end of the phrase, it is forced to move down the half-step to the 16th because the clarinet and cello have come to rest on the 22nd and 6th. The result is that the 16th, which, being closely related, pulls, in our minds, on the 17th, which is unrelated, and forces the resolution. This idea is worth looking into—I certainly mean to explore it further.

Other aspects of this idea of resultants would be interesting to consider. I wonder, for example, whether some such perception might not have been at the root of Schenker's intuition. (This suggests another approach to the problem of extending it past the merely triadic.) But I should think that what I have written here already starts enough hares for chasing down.

Fig. 26. “The Chief's Speech,” mm. 33-51. Accompaniment only. The spoken foregroung parts have been omitted.

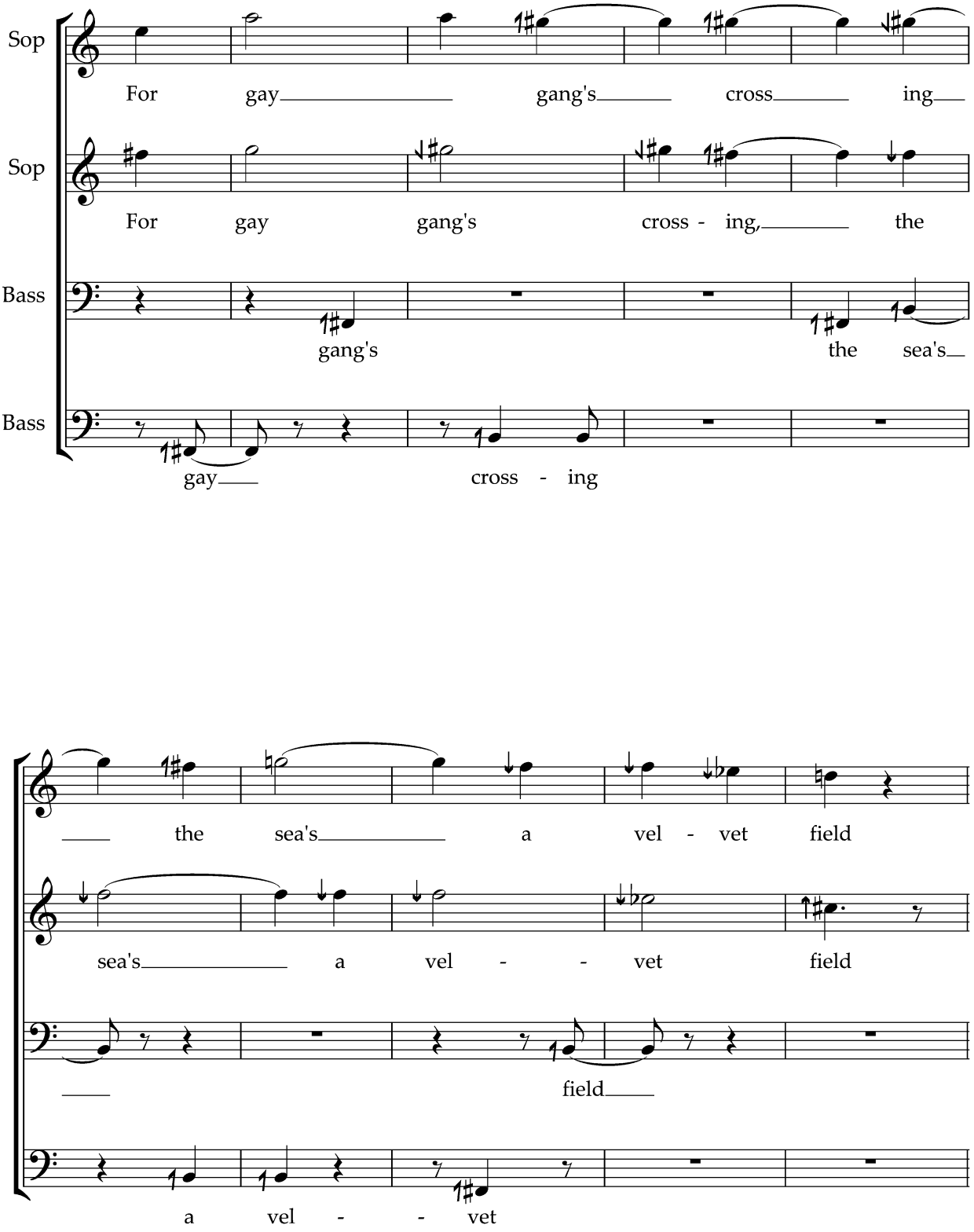


Fig. 27a. The scale in use in mm. 33-51.

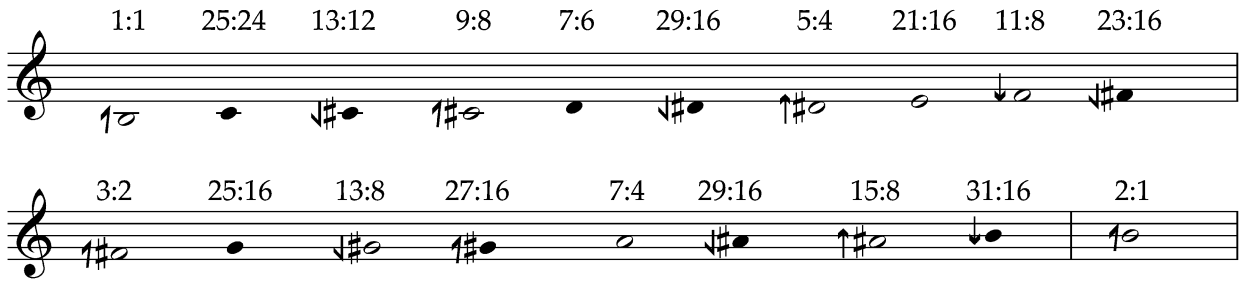


Fig. 27b. The notes of Fig. 26 labelled with their harmonic numbers showing the difference tone relation of the bass to the sopranos.

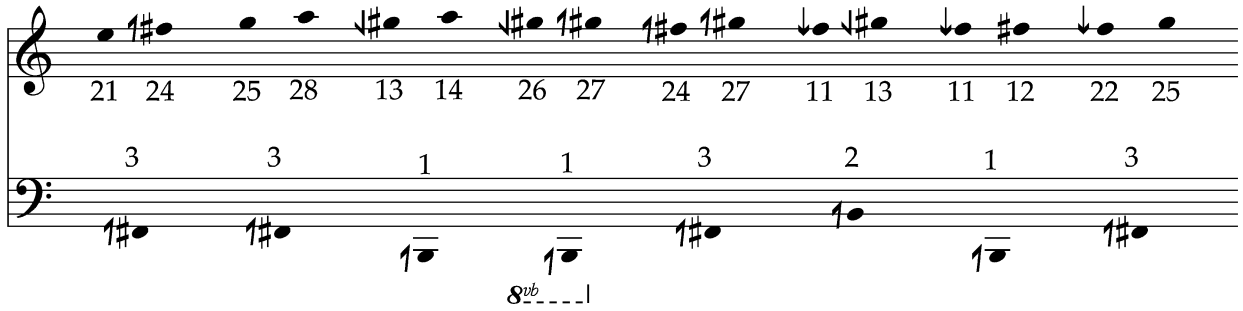


Fig. 28. "The Chief's Speech," mm. 51-63. Accompaniment only.

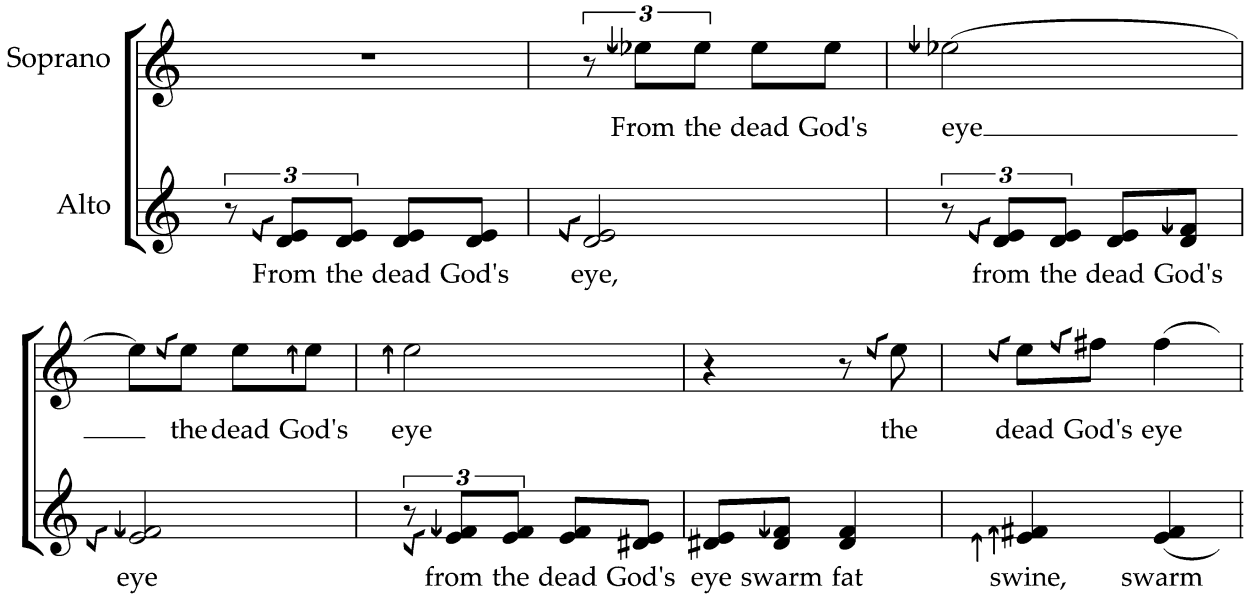


Fig. 29a. The scale in used in mm. 51-63.

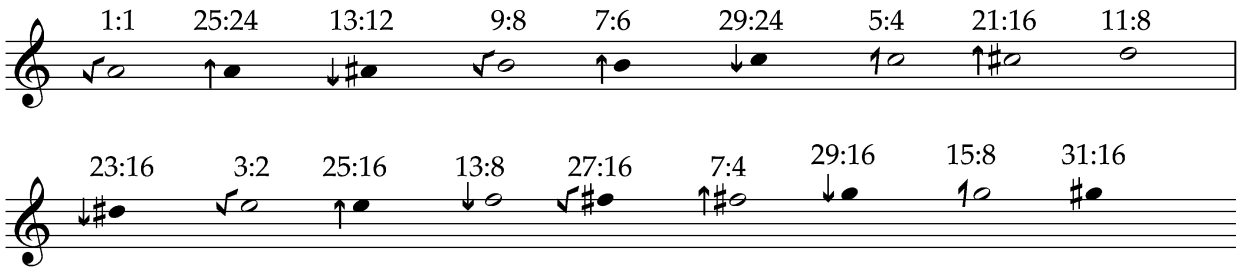


Fig. 29b. The notes of Fig. 26, labelled with their harmonic numbers, as in Fig. 27, showing the soprano's notes as summation tones of the altos'.

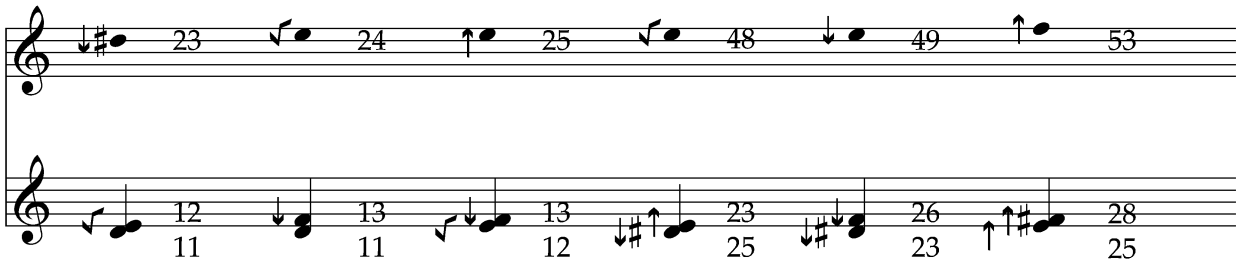


Fig. 30. “The Chief's Speech,” mm. 64-75. Accompaniment only.

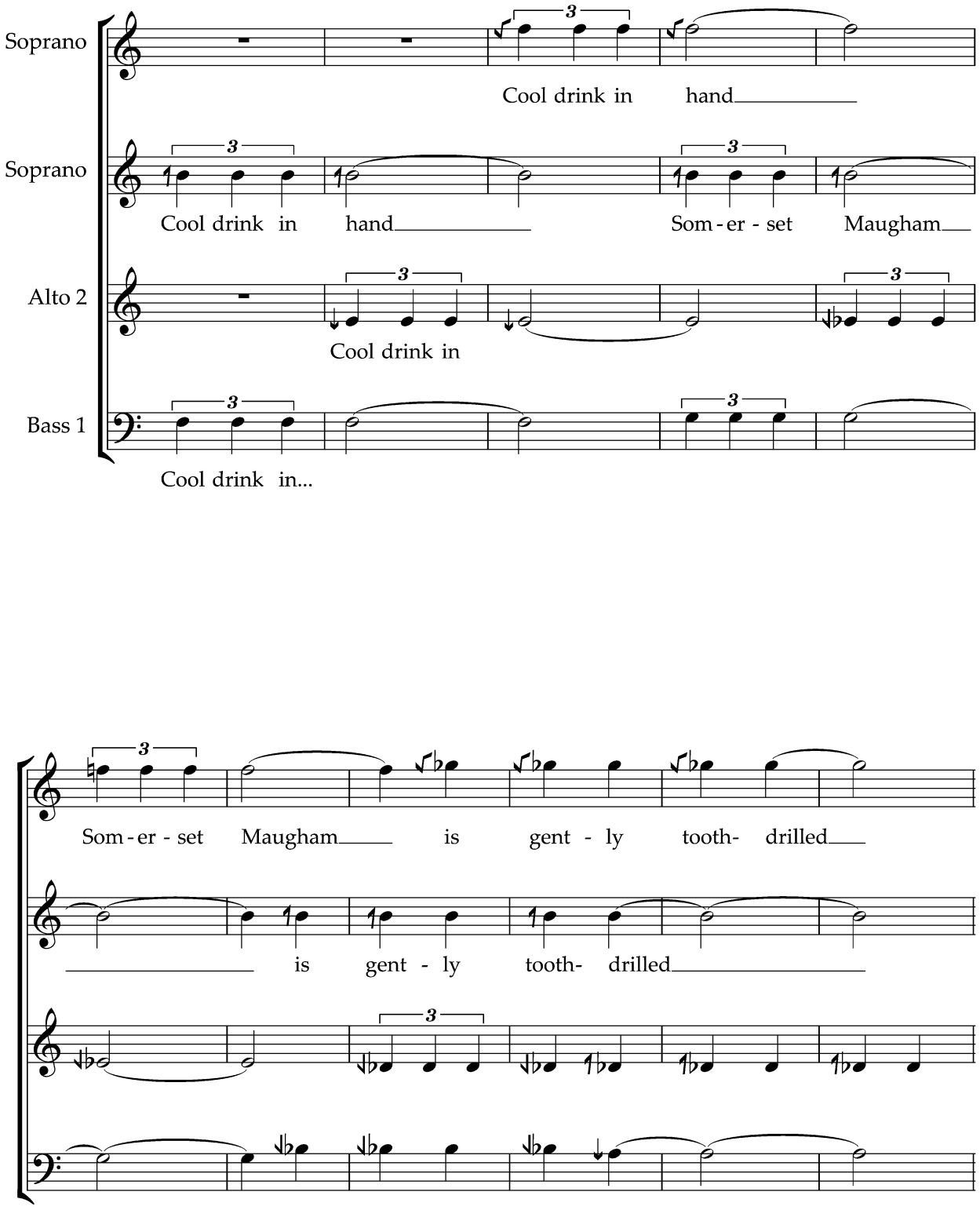


Fig 31a. The scale in use in mm. 64-75.

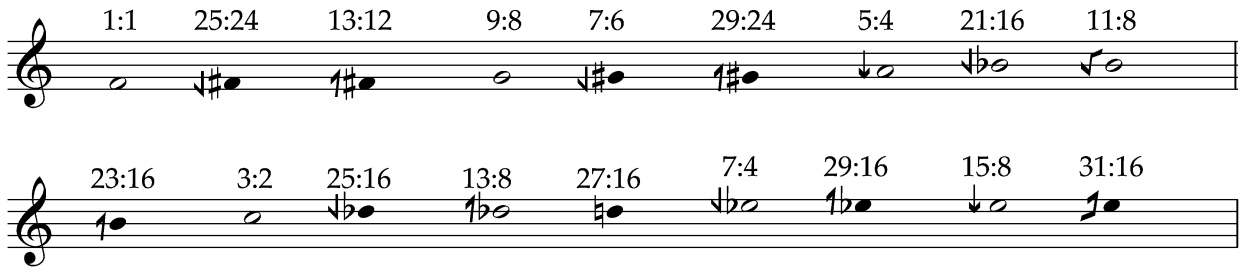


Fig. 31b. The notes of Fig. 30, labelled as in Fig. 27, showing the tenor's notes to be the difference tones, and the soprano's, the summation, of the notes of the alto and brass(?).