"MAXIMUM CLARITY"

AND OTHER WRITINGS ON MUSIC

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It is now possible to say that the concept of tonality has been elucidated and generalized. Using just intonation of various kinds, it can be shown how traditional seven-tone triadic tonality expands to twelve-tone chromaticism and thence to scales of fifteen, nineteen, thirty-one, thirty-four, fifty-three, and sixty-five tones per octave (plus an indefinitely great number of still more numerous scales). Using this system as a model, alternative tonalities based on analogous but different triads, tetrads, pentads, or hexads (or still more numerous basic chords) can be constructed.

To have a tonal system with \( n \) notes in its basic chord \((n\text{-th ad})\), you need \( n \) prime numbers greater than 1. Normally 2 is the cycling number, since most scales extend to the compass of an octave and repeat their patterns in all octaves so that every note has octave equivalents. It would be theoretically possible to use 3 or any other prime number as the cycling interval. (In the case of 3 as cycling number, scale patterns would extend over the interval of a perfect twelfth, and each note would have twelfth “equivalents.”)

Three is normally the tonal number when it is included in a system. The interval \( 3/2 \) (perfect fifth) provides the relation between tonic and dominant or between subdominant and tonic. Progressions analogous to these are traditionally called tonal progressions.

In a traditional triadic system, the prime numbers greater than 1 are 2, 3, and 5. Five (referring to major and minor thirds and sixths) is a modal number, traditionally so-called because it affects major–minor coloration. Progressions where the root moves a third or a sixth are called third-related (sometimes tertiary) progressions.

If the seventh or eleventh partials of the overtone series are introduced, they constitute new modal numbers, providing additional coloration in the harmony and the progressions. If any higher primes are present when 3 is not, normally the smallest of these becomes the tonal number. It is not possible to make any prime number the tonal number. In such a case, other primes greater than 2 become modal numbers and effect changes of chord color.

The Pythagorean (also medieval Western and ancient Chinese) system is a diadic system. Its basic chords are perfect fourths and fifths. Strictly speaking such a system provides no modal coloration. Pythagorean thirds and sixths can form triads in a distorted imitation of true triadic harmony, but normally these intervals are considered dissonances, since they are actually distant tonal notes rather than bona fide modal intervals.

In order to express graphically the harmonic relations possible in a given system, one needs an \( n \)-dimensional lattice of ratios, each ratio representing a note in the system. On each principal axis of the lattice, a particular prime and its powers are compared to powers of the cycling number.

Usable tonal scales are composed of neighboring ratios in a lattice, the ratios rearranged to be in ascending order of magnitude. Each acceptable scale in a system has the same number of discrete adjacency ratios as there are notes in the basic chord, one for each prime number greater than 1. This is one more than the number of axes in the lattice.

A basic major chord consists of a point in the lattice and one point in a positive direction on each of the axes. The parallel minor chord uses the same notes on the tonal axis and one step in a negative direction on the modal axis. This step is taken from the ratio one step farther in a positive direction on the tonal axis than the starting point.

The first set of scales to be derived are diatonic major scales; the second set, parallel diatonic minor scales; the third and last set, chromatic and hyperchromatic scales. The chromatic scale is composed of the mixture of parallel major and minor diatonic scales plus whatever adjacent ratios are necessary to produce an acceptable scale. Hyperchromatic scales involve choosing additional notes in all directions in the lattice, rather than restricting choices in modal directions.

The notation assigns the note names C, D, E, F, G, A, B to the major diatonic scale of the traditional triadic system. The symbols + and - show the raising or lowering of notes by the syntonic comma \((81/80)\), which is about 22 cents in size (approximately a 1/10th tone). Each new prime number introduces a new pair of inectional ratio symbols. Prime number 7 introduces \( \times \) and \( \div \), which indicate raising or lowering a pitch by 36/35, or about 49 cents (approximately a quartertone). Prime number 11 introduces up-arrow and down-arrow, \( \uparrow \) and \( \downarrow \), which indicate raising or lowering a pitch by 33/32 (somewhat more than a quartertone).  \( ^2 \)
Scalar systems can now be derived by computer, using a program in Fortran IV which was written by Ed Kobrin. An upcoming issue of Source: Music of the Avant-Garde will present this program along with all scalar systems involving prime numbers up through 11. The one involving 2, 3, 5, 7, and 11 entails using a four-dimensional lattice. The rules for deriving scalar systems can be understood from a study of the flow chart of this program.

The lattice of the 2, 3, 5 system, given in Figures 20 and 21, yields the scales given in Figure 22. Exactly analogous systems of scales may be obtained with other selections of prime numbers. A number of such systems will appear

\[
\begin{align*}
&2 & &3 & &5 & &9 & &9 & &Bb & &D \\
&1 & &5 & &2 & &6 & &3 & &Eb & &G \\
&1 & &4 & &5 & &8 & &1 & &Ab & &C \\
&3 & &5 & &2 & &4 & &3 & &F & &A \\
\end{align*}
\]

7-tone diatonic major

\[
\begin{align*}
&36 & &9 & &9 & &8 & &5 & &2 & &5 \\
&6 & &3 & &15 & &2 & &8 & &1 & &4 \\
&8 & &1 & &5 & &4 & &3 & &24 & &24 \\
\end{align*}
\]

12-tone chromatic

**Figure 20.** Lattice representations of diatonic major, minor, and chromatic scales of traditional triadic tonality. Three-limit relationships (i.e., by chains of perfect fifths) are mapped on the vertical axes, 5-limit relationships (i.e., by chains of major thirds) on the horizontal axes.

\[
\begin{align*}
&243 & &243 & &50625 \\
&200 & &160 & &32768 \\
&81 & &81 & &16384 \\
&80 & &64 & &65536 \\
&225 & &225 & &262144 \\
&1125 & &1125 & &5625 \\
&625 & &25 & &64 \\
&125 & &25 & &16 \\
&125 & &25 & &16 \\
&175 & &25 & &16 \\
&2048 & &2048 & &56 \\
&8125 & &8125 & &56 \\
&32768 & &32768 & &56 \\
&16384 & &16384 & &56 \\
\end{align*}
\]

**Figure 21.** Lattice of the sixty-five-tone hyperchromatic tridic system (using prime numbers 2, 3, and 5), expressed in ratios (above) and in letter names (below). This is a two-dimensional lattice, showing the 3-axis vertically and the 5-axis horizontally; the 2-axis is omitted in this diagram and in all the other lattice diagrams in this book, as it merely shows 2/1 relationships (octaves) of the pitches in the other dimensions.

in the Source issue mentioned in the previous paragraph, under the title "Phase 1-B."

The foregoing is a description of Phase 1 of a project undertaken by Ed Kobrin and me. Phase 2 will consist of computer programs aimed to generate compositional decisions based in part upon the scale systems of Phase 1.
FIGURE 22. Scales within a 5-limit just intonation system, culminating in a sixty-five-tone hyperchromatic scale.
These partly generate compositions in which the parameter of harmony is
germane, as it is not in, for example, most serial or atonal compositions.
Alternatively, one can say that consonance–dissonance becomes once more a
significant compositional variable.

Phase 3 involves the design of an interface between a small computer and
musical synthesizer equipment. The aim is to make possible real-time inter-
action between performers and the computer. A PDP-5 computer was don-
ated for our use by Digital Equipment Corporation. The rest of the hard-
ware is designed and built by Ed Kobra. The computer will act not only as an
instrument but also as a decision maker in the composition process. It
will function onstage with performers and their traditional instruments. The
contributions of the composer will have been made in preparing the available
software (programs) for the computer, based upon Phases 1 and 2. We are
aiming at a first composition, a commission from violinist Paul Zukovsky,
using these means.4

As we near the completion of Phases 2 and 3, we plan a complete report
on the project, which we have promised to Perspectives of New Music.

The project raises questions not only about the use of “artificial intelli-
gence” in the production of music, but also about the nature of tonality
and related questions of pitch usage in music.

NOTES

1. [Ed.: A gloss on this complex paragraph may perhaps be helpful. The “basic
major chord” Johnston refers to is, in ratio terms, 1/1–5/4–3/2, which “consists of a
point in the lattice” (1/1) and “one point in a positive direction on each of the axes,”
i.e., the 3–axis (giving 3/2) and the 5–axis (giving 5/4). The “parallel minor chord”
i.e., the tonic minor, in ratio terms 1/1–6/5–3/2) is made up of “the same notes on the
tonal axis” (the tonal axis being the 3–axis, as defined in the text), hence 1/1 and
3/2, and the pitch that lies “one step in a negative direction on the modal axis” (in
this case the 5–axis) from “the ratio one step farther in a positive direction than the
starting point” (namely 3/2, the starting point being 1/1), i.e., 6/5.]

2. [Ed.: When Johnston writes that the symbols 2 and 7 “indicate raising or lowering
a pitch by 36/35, or about 49 cents,” he is taking the just (5–limit) major scale
as his starting point: for example, 36/35 is the amount by which 9/5 (the just minor
seventh) must be lowered to give the 7/4 (the “seventh” minor seventh).]

3. [Ed.: The article “Phase 1–B” was never completed. Much of the intended
content appears in more developed form in “Rational Structure in Music.”]

4. [Ed.: Unfortunately, the violin piece for Paul Zukovsky was never realized.]