"MAXIMUM CLARITY"
AND OTHER WRITINGS ON MUSIC

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UNIVERSITY OF ILLINOIS PRESS
Urbana and Chicago
“Over the whole of the historical period of instrumental music, Western music has based itself upon an acoustical lie. In our time this lie—that the normal musical ear hears twelve equal intervals within the span of an octave—has led to the impoverishment of pitch usage in our music.”

We lie especially when we pretend to ourselves that vertical combinations of these pitches constitute harmony. We do not avoid the lie if we abandon harmony in music, so long as we retain a tempered scale.

Feeling that the harmonic mode of pitch perception is far too important a resource of human capability for it to be allowed to fall into disuse, I have set about to reestablish ratio scale usage in pitch organization. This has entailed a number of radical means (large numbers of microtones, for instance, entailing new performing techniques, especially for wind players), some strongly conservative practices such as the resumption of a sharp awareness of degrees of consonance and dissonance as a major musical parameter (which amounts to revoking Schoenberg’s much-touted “emancipation of the dissonance”), and even some radical reactionary attitudes, for example, the rejection of the idea that noise, “randomness,” and ultracomplex pitch are the primary frontiers for avant-garde exploration.

I have been concerned to reopen doors closed by the acceptance of the twelve-tone equal-tempered scale as the norm of pitch usage. My focus is upon complexity arrived at as perceptible order rather than as seeming disorder. I am especially interested in the role played by proportional order in the domains of rhythm, harmony, and melody. Since as a culture we have developed not only a tolerance but also a taste for a high level of complexity in many areas of experience, this effort necessarily involves a very highly developed proportional ordering system in order to cope with such levels of complexity in pitch and rhythm. We must learn to differentiate sharply between complexity due to large numbers and complexity which delineates subtleties of relationship.

If one purifies Western pitch usage of late adulterations traceable to the adoption of equal temperament, the two most complex examples of systems of proportional relations in existence are the harmonic and metrical systems of Western art music. Rhythm requires no corrective, only the long-since-achieved abolition of the tyranny of the bar line and the fixed repetitive metrical schemes proper to common-practice-period music.

Extrapolating the basic logic from the harmonic practice of traditional Western music is only a first step. A much more challenging and interesting follow-up is the generalization of this logic so that it becomes applicable to unfamiliar pitch materials. Once this has been achieved, the door closed by the acceptance of equal temperament is reopened.

In measurable physical terms, there are only two parameters of sound, duration and energy (measurable as the amount of displacement of molecular particles in a vibrating medium). These are interpreted phenomenologically as a great variety of parameters, a differentiation assisted enormously by our facility in perceiving gestalt phenomena on various scales of time.

On an ordinary scale of time (countable time), the patterning, due always to vibratory events, is intelligible as rhythm. As we move up into larger amounts of time, our ability to count diminishes and the role played by memory greatly increases. Macrorhythmic events, characterizing longer durations, enable us to acquire a sense of the overall shape of music or sound compositions. We also perceive patterning on a scale of time much too rapid to count, but clearly we have nevertheless a marvelously precise ability to measure the duration patterns involved in these microrhythmic events.

Since air or any other vibrating medium can respond to the periodicities of diverse sound sources simultaneously, these different rates of vibration are constantly interacting with one another. If the pattern of interaction is relatively simple, the sound complex exhibits a type of blend traditionally called consonant (as defined in, for instance, Helmholtz’s On the Sensations of Tone). This kind of consonance, though not independent of context and function in a melodic or harmonic pattern, does exhibit, simply in its blend, a greater simplicity than the concomitant kind of dissonance, in the same way that superimposed rhythmic groupings such as 3:2 and 4:3 are clearly easier to perform and to analyze by ear than, say, 9:5 or 15:8.

We also identify such combinations by their relative “size,” as measured for instance in half steps or in cents. Because of this double classification of musical intervals (relative consonance and relative size), an interesting situation occurs when there is little difference in size between two intervals, yet
a markedly audible difference in consonance. A small increase in the rate of vibrations of the upper frequency of a just major third (the kind found between the fourth and fifth partials of the overtone series: ratio 5/4)—enough to increase the number of vibrations of the upper frequency from 80 per 64 of the lower frequency to 81 per 64—will produce a marked difference in roughness of blend. Since $81/64 = 3/2 \times 3/4 \times 3/2 \times 3/4$, this interval can be arrived at by tuning a frequency a perfect fifth ($3/2$) up from the lower frequency, then another a perfect fourth ($4/3$) down from that one, and then repeating this pair of tunings. Such tuning sequences abound in traditional musical practice, so the $81/64$ major third, called a Pythagorean ditone, is actually a relation used in much untempered music.

Our habit of detuning all the intervals of our scale except the octaves in order to render it an equal interval scale makes us less sensitive to such differences, an insensitivity which we compel ourselves to accept as a constant norm. Because we have selected an “average” size for all intervals near the size of a major third, we use neither the simplest vibration ratio near this size ($5/4$), nor any of the more complex dissonant ones which are the result of whole-number relations (e.g., $9/7, 32/25, 11/9, 14/11, 81/64$). Instead we use the cube root of 2 for the tempered major third, which renders its harmonic meaning (its potential consonant or dissonant relation to other interval combinations in the context, which is expressible in terms of rational fractions) entirely ambiguous. For purposes of systematic harmony we would desire an unequal interval scale were it not for our habitual desensitization. This is in conflict with the natural desire not to have to cope, melodically, with, for instance, several sizes of whole steps and of half steps, a problem that arises in performing chromatic passages in Baroque music.

One of the results of this state of affairs has been, after about a century of such a norm of pitch usage, the loss of harmonic (simultaneous) pitch relations as an effective organizing device in much of the serious art music of our culture. A less obvious but, I think, more serious result is that what had been an open, evolving pitch system (developing within notated musical history in Europe from a harmonic norm of perfect fifths and fourths to a norm of major and minor triads) became a closed system capable of being exhausted of fresh, permutational possibilities of its available interval combinations. As a result, on the level of small-scale rhythmic context, possible new configurations are eventually used up entirely, creating a predictability of vertical (and “diagonal”) combinations. This predictability is unrelieved, in contrast to the case of predominantly triadic music, by any larger rational system of harmony which provides a sense of movement and change within an ampler context. Efforts to create such a harmonic system on the basis of serial order have not met with widespread listener understanding.

It seems therefore that a primary effect of the celebrated “emancipation of the dissonance” has been to deprive music of one of its most effective means for organizing longer spans of time. This has been a process long in the making. It began innocently enough with a simple neglect: music notation did not attempt to incorporate into its symbols pitch differences of a syntonic comma (ca. 21.5 cents, or ca. 1/5 of a semitone) resulting in performance practice from a shift, in compositional practice during the fourteenth century AD, from the consonances of organum to those of major and minor triads; and in response to the challenges posed by the emergence of an independent repertory and practice of instrumental music, it culminated in the acceptance of keyboard temperaments in the sixteenth and seventeenth centuries. This last process impoverished the logic of harmony and tonality by weakening the perceptibility of consonance and dissonance. This impoverishment, right from the outset of tonal tradition, resulted in loss of the support which contextual functional relations can gain from a perceptible mathematical logic reflecting the hierarchy of the system of harmony and tonality.

The frequently cited gain in ambiguity and flexibility hardly seems comparable to this loss. It was, in fact, not this aspect of temperament which first recommended its adoption, but the practical solution to the problem of tuning keyboard, fretted, and other fixed-pitch instruments for a music that implied a wide harmonic range. Once equal temperament with twelve tones—the optimum compromise for modulatory triadic music—was adopted, it took only a little over a century for the exhaustion of the fresh possibilities of the system to become a major aesthetic problem.

It has been useful in acquiring mastery of this complex problem to seek, in collaboration with Edward Kobrin and Peter Rumbold, two computer programmers who are also skilled composers, a single algorithm which would enable a computer to derive both the Pythagorean scale system and the triadic scale system known as just intonation. Each scale system is infinitely expandable, to scales with greater and greater numbers of notes per octave. Thus not only the rules for deriving pitches were needed, but also rules for determining when a collection of pitches constitutes an acceptable
ratio scale. Interesting differences between the two scale systems had to be explicable as solely the effect of greater and less numerical complexity in their constituent intervals (ratios). Lastly, the algorithm thus derived has to be applied to unfamiliar numerical combinations, to produce as yet unexplored scale systems, and these systems tested out as the basis of musical compositions.

This work is at present far advanced, and it is possible now to present scale systems involving not only prime numbers 2 and 3 (Pythagorean), or 2, 3, and 5 (triadic just intonation), but all combinations of prime numbers no larger than 11.

The first presentation of any of this material, an article in *Source* 7, "Phase 1–8," by Edward Kohn and me, gave only "one-dimensional" scales and accompanying matrices (that is, scales derived from using 2 and one other prime number). The program, in Fortran IV, was also published.

Later, in volume 6 of *Proceedings of the American Society of University Composers*, I published the derivation of the triadic just intonation scale system, this time without a computer program, in an article called "Tonal-ity Regained." This scale system, involving two prime numbers (3 and 5) greater than 2, is a two-dimensional scale system. I had already, in my String Quartet no. 2 and the orchestral work *Quintet for Groups*, used microtonal ultrachromatic scales from this tuning system.

In 1971 I began a series of compositions using unfamiliar tuning systems but not requiring unusual instruments or electronic means. The choral works *Rose* and *Mass* use "diatonic scales" from systems based upon 2, 3, and 7, and 2, 3, 5, and 7. My String Quartet no. 4, commissioned by the Fine Arts Quartet, progresses from Pythagorean (2, 3) tuning through just intonation (2, 3, 5) to a tuning based on 2, 3, 5, 7. It goes as far as a twenty-two-tone chromatic scale in this system. One of the insights this experience has brought me is that precision in just tuning is the more crucial the more notes per octave one uses. This is above all because more than ever, the harmonic and melodic structural functions need the underpinning that greater clarity of differentiation in tuning provides.

The success of these works has greatly encouraged me to continue my research. Parallel to this research has been an application of its findings in the field of metrical rhythm. The most highly developed examples are my *Knocking Piece* for two percussionists and piano interior, *Quintet for Groups*, *Sonata for Microtonal Piano* (also in ultrachromatic triadic just intonation), and my String Quartet no. 4.

While the works involving extended triadic just intonation emphasized new and sometimes starting pitch relations, those involving unfamiliar tuning systems up to now emphasize "normal" and "logical-sounding" pitch combinations. Part of the explanation for this lies in the wisdom of walking before trying to learn to run; but an equally important consideration has been the wish to recapture the beauty of intelligibility and simplicity in an unfamiliar guise.

The most rudimentary mode of perceiving a very complex sound gestalt merely tags it as unique. It is identifiable as itself, should it recur, and can be differentiated from other gestalts.

As soon as we want to compare similar events, we must single out qualities in terms of which to make such comparisons, and we are involved with various perceptual parameters. So long as our measurement of a parameter consists only of determination of greater or less on some gradual scale between perceptually linked pairs of opposites (such as high and low, loud and soft) we are dealing with rough contours merely.

For more precise comparisons or descriptions, we must quantize these parameters and try to assign positions on an imaginary linear scale to given perceptual values. It is here that, for instance, melodic perception of pitch contours becomes meaningful. It is also here that serial organization has meaning.

When simultaneous periodic phenomena such as frequencies (or on an ordinary time scale, tempos) are considered, a further ability emerges: the quantitative comparison of amounts of time, yielding a precise sense of proportion. On the scale of ordinary time, this perception gives us metrical organization. In microtime, the comparison of rates of vibration gives us harmony, a subjective reaction possible because of our unconscious but precise measurement of ratios between frequencies. For this mode of perception, the equal-interval scales adequate for melodic perception are not at all precise enough.

All musical pitch scales are compromises between these two perceptual modes of processing sound, the melodic and the harmonic. For melodic purposes, we need to know which of two pitches is higher, and how much higher. It is convenient to measure such distances additively, as if they were ranged on a linear scale measured in equal increments.

The minute we ask what increments we will use, we are plunged into another perceptual mode, the harmonic or proportional one. Ubiquitously, human cultures divide up the pitch continuum into octaves. An octave is
defined physically as the frequency ratio 2/1. And as soon as we want to know not only if a given pitch interval is greater in linear size than another, but also if its blend, its consonance, in Helmholtz’s use of the term, is smoother or rougher, we are similarly involved in harmonic listening.

Harmonic relations are not conveniently thought of additively, but rather multiplicatively. What we think of melodically as adding two intervals is, from the standpoint of their pitch ratios, multiplying them. “A major third plus a minor sixth equals an octave” translates to “3/5 multiplied by 8/5 equals 2/1, or 3/4 x 8/5 = 2/1.”

In using a ratio scale, both melodic and harmonic considerations enter. For this reason, two arrangements of ratios (or their equivalent note names) are useful: the n-dimensional lattice and its linear projection into a scale arranging the pitches in order from lowest to highest, within the span of a single octave.

The lattice demonstrates harmonic neighbors (that is, ratios in near proximity are consonant, those farther away being dissonant). The linear scale demonstrates melodic neighborhoods, a concept already familiar from common use.

The lattice shows not only that two ratios are consonant or even how dissonant they are in comparison to others, but by what chain of relationships their dissonance is “explained.” To “explain” a dissonance in this manner justifies it to the ear; it is heard as a natural and inevitable result of a constellation of less complex ratios.

The linear scale not only provides a model for scalar melodic passages, but also a schematic for nonadjacent design elements which form nevertheless a scalar armature for melodic lines (as in Schenker’s Urlinear). Patterns or “paths” in the lattice may be used to give a sense of harmonic direction even to passages that lack entirely any conventional tonal or harmonic “logic."

Scale derivation is thus not simply a theorist’s justification ex post facto of a particular way of conceiving music. It is also not merely a device for generating new arbitrary “systems” for experimenters to play with. Rather it is an effective aid in designing melodic and harmonic audible structure even with unfamiliar pitch materials. Such structure is not serial in type, though serial patterning does indeed result in the interval sequences which comprise the adjacency relationships of the linear ratio scales in all systems of the type described. One may speak of tonality, but only if the term is generalized in such a way that it is clear that triadic tonality (whether diatonic or chromatic or ultrachromatic) is only one of many possible types of harmonic (ratio scale) logic. Similarly, modality recedes in importance to become only a particular way of using the materials of a given tonal system, just as is modulatory harmony.

A great advantage which ratio scales have over all systems of temperament is that most of the latter are mutually exclusive, and the permutations of available intervals of each are finite and exhaustible, whereas all ratio scales are subsets of larger sets and are infinitely relatable and expandable.

There is in this situation something of the embarrassment of riches. At first it may appear that “all is possible, all is meaningless.” But in fact it is not at all true that an infinite system renders all possibilities either available or equal in significance. Also, even a cursory consideration of method will render clear that one is always necessarily dealing at any time only with a subset or some subsets of the infinite set.

It may be objected that a ratio system is hierarchical and thus philosophically inferior to a system in which there is a total democracy of elements (even assuming that such a system is actually possible). I will counter that an organism is also hierarchically ordered, and that any success in reducing the interrelations between its parts to a total or even approximate democracy will result in its death. We do, in other words, deal necessarily with hierarchical order, which is appropriate and necessary to certain important kinds of functioning. Whether aesthetic order is of such a kind in all cases, or in all successful cases, is a point I do not wish to argue. Suffice it to say merely that hierarchical order is at least one viable kind of aesthetic order."
the mathematical model, either numerator or denominator of any ratio may be multiplied or divided by any power of 2 without changing the musical meaning of the expression. By custom, all ratios are "octaved" in this manner to bring their value between 1/1 and 2/1. This has the practical effect of transposing all ratios by octave into the span of a single octave, bounded by 1/1 and 2/1. Thus all ratios are customarily expressed as improper fractions smaller than 2/1. This greatly simplifies the formation of linear scales, since the only further step is to sort out the ratios into ascending order of magnitude. The magnitude is measured by a logarithmic quantity, typically the cent (1,200 per octave). Other measurements may be used, for instance the savart (301 per octave). A simpler way to compare relative sizes of ratios is to convert them into decimals.

In constructing a ratio lattice, it is convenient to lay out the axes first and then, using the same technique used in matrix formation, to fill in the intermediate points coordinating all points on all axes. The lattice will comprise a plane, a solid, or a hypersolid, depending upon the number of axes used.

Any scale system with more than one prime number greater than 2, in order to be of the same type as the diatonic–chromatic–ultrachromatic system of triadic just intonation scales (which uses 2, 3, and 5), will have less numerous parallel "major" and "minor" scales which are then combined and filled out to make chromatic and ultrachromatic scales that are more numerous (Figures 26, 27, and 28). The difference between diatonic and chromatic scales is created by a special restriction on diatonic (major and minor) scales. In these scales, all ratios are chosen from a set of columns parallel to the axis associated with the smallest prime (called the tonal axis) and one step in either a "positive" or a "negative" direction on each of the other axes (the modal axes). The "positive" direction generates major scales; the "negative," minor scales. The formation of these two sets of scales is exactly analogous and parallel. The minor is the inverse of the major, the symmetry being organized around a pair of ratios adjacent on the tonal axis, the center, 1/1, (zero point), and the ratio one step in a positive direction on the tonal axis. The point at which diatonic major and minor scales are combined to form

**FIGURE 23.** The 2, 3, 5 pitch lattice (the 3 and 5 axes only are shown, the 3-axis vertical, the 5-axis horizontal).

**FIGURE 24.** The 2, 3, 7 lattice (the 3 and 7 axes only are shown, the 3-axis vertical, the 7-axis horizontal).
FIGURE 25. The 2, 3, 5, 7 lattice (the 3, 5, and 7 axes are shown, all at right angles to each other).

the chromatic scale is arbitrary. I have set it as the first point when the largest linear scale adjacency ratio is smaller than 250 cents (in the case of a two-dimensional lattice), than 125 cents (in the case of a three-dimensional lattice), than 62.5 cents (in the case of a four-dimensional lattice), etc. In chromatic and ultrachromatic scales, the ratios are drawn from all directions in the lattice.

An acceptable ratio scale must have as few different intervals between its adjacent notes as possible—no more than the quantity of prime numbers in use (including 2). Each scale derivation begins with the “basic chord of the system,” either major or minor. This consists of the zero point and one adjacent ratio on each of the axes, in either a positive (major) or negative (minor) direction. Each time a ratio is tested to be added to the scale, it must satisfy various conditions. It must be adjacent in the lattice to a ratio already in the scale. It must divide in two the largest adjacency ratio in the scale already selected. If there are other adjacency ratios in the scale the same size as the one to be divided, each must be divided analogously or in exactly the inverse manner. One of the new adjacency ratios obtained by this division must be identical to one already present between neighboring scale ratios. If there is a choice of ratios which satisfy these conditions, the one that divides the largest adjacency ratio most nearly evenly (as measured in cents or some other logarithmic quantity) is chosen. If there is a choice of such ratios, one selects the one nearest the zero point or nearest the ratio one step in a “posi-
**Figure 27.** Scales derived from the $2, 3, 7$ lattice, the last being a chromatic scale very different from that in Figure 26.

tive" direction on the tonal axis. This distance is measured in steps along or parallel to one of the axes. If two usable ratios are equidistant, a ratio on an axis associated with a smaller prime is preferred. A ratio in a sector where more factors cancel out when the axis coordinates are multiplied is preferred over one in which fewer factors cancel. If the largest adjacency ratio cannot be acceptably divided by any of the available ratios, the next largest should be considered.

As long as the scale is still diatonic, only ratios may be used which lie on the tonal axis or in the columns adjacent to it one step in either a positive direction (for major) or a negative one (for minor).

When the major and the minor diatonic scales are combined to obtain the

**Figure 28.** Scales derived from the $2, 3, 5, 7$ lattice, the last being a twenty-two-note chromatic scale.

chromatic, all identical adjacency ratios which are divided must be so either in the same or in an inverse manner in all cases. If in two or more cases such an adjacency ratio is divided differently, all cases must utilize simultaneously all such divisions, resulting in two or more interpolated ratios. If, when these conditions are satisfied, the scale still has too many different adjacency ratios, a ratio or ratios must be found which divide any adjacency interval.
(starting with the largest, as preference) in such a way as to introduce two new adjacency ratios both of which are already present between scale ratios. If after this effort the scale still has too many adjacency ratios, all the possible divisions of all adjacency ratios must be examined to find cases where common occurrences of new adjacency ratios make it possible to reduce the total of adjacency ratios to the minimum level. In some cases this necessitates a large number of scale ratios for the smallest acceptable chromatic scale in a given scale system.

It will be obvious that this process is easier to carry out with the assistance of a computer. Nonetheless, in the process of generating the program to facilitate finding these scales, it was necessary for me to derive all the scales which were to be made available by means of the program before it existed in order to be sure of covering all eventualities. It may seem, therefore, that producing the program achieved nothing. This is assuredly not so, since it was the pressure of having to be so extremely specific and inclusive that made possible my solution to these theoretical problems. I needed the program not mainly for derivation, but in order to make possible the study of reasoning processes necessary to translate an intellectual process which is at least partly holistic in character entirely into a binary language.

NOTES

[Ed.: “Rational Structure in Music” was first presented as a lecture for the American Society of University Composers in 1976 and delivered again on several occasions thereafter, including in Bonn and in Paris in 1981. In the lecture Johnston used occasional taped examples, played on a Motorola Scalatron organ, of intervals and chords in just intonation. It has not seemed useful to reproduce these examples here in notated form.]

1. Quoted from the composer’s remarks in Abram Lofy’s notes to the Fine Arts Quartet’s second performance (Chicago, April 28, 1974) of String Quartet no. 4.


3. [Ed.: At this point in the original lecture, Johnston incorporated a taped performance of his choral work Rose performed by the University of Illinois Summer Chorus, conductor Neely Bruce.]

4. [Ed.: At this point in the original lecture, Johnston incorporated a taped performance of his String Quartet no. 4 by the Fine Arts Quartet, recorded at Wilmette, Illinois, April 29, 1974, by radio station WFMT, Chicago.]

A NOTATION SYSTEM FOR EXTENDED JUST INTONATION

2003

Extended just intonation is a kind of ratio-scale notation for musical intervals—i.e., those analogous to the pitch ratio series. To handle the complex ratio relations necessary for accurate just tuning of modulatory triadic music, and for extensions beyond simple triads, a basic notation that is familiar, widely used Western music notation is designed. The fundamental differences in what the symbols refer to are simply that there is no ambiguity in such a procedure.

This notation is not tied to any particular diatonic scale. It is based on the ratio 435 Hz, or even a C-based or G-based tuning of the instrument. The constant is the ratio relations between pitches. The accidentals are the ratios of the notes to C, since the key signature of C major is such that the notes of the scale are related to C by the ratios of the scale notes. The ratio relationship to C of the scale notes is 1/1 (F), 3/2 (G), 5/3 (A), 15/8 (B), 2/1 (C). The sequence is 9/8, 10/9, 16/15, 9/8, 10/9, 9/8, 16/15 (Figure 29).

This results in a scale in which there are two sets of intervals: a half step and one size of half step (16/15). The intervals are divided into two kinds of whole step is 81/80, the comma of which is represented in notation by the accidentals + (raised) and − (flattened). The interval, C, D into a 10/9, it must be notated C, C, the interval D, E into a 9/8, it must be notated D, E++ or E++.

In examining this just major scale for its further use, we must keep in mind that a just minor triad is the exact opposite of a major triad. To add intervals (e.g., C, E plus E, G equals the ratios (correspondingly, 5/4 multiplied by 6/5 etc.)