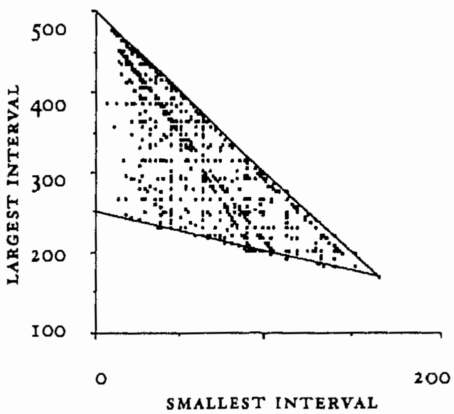
9 The Catalog of tetrachords

THIS CATALOG ATTEMPTS a complete and definitive compilation of all the tetrachords described in the literature and those that can be generated by the straightforward application of the arithmetic and geometric concepts described in the previous chapters. While the first of these goals can be achieved in principle, the second illustrates Aristoxenos's tenet that the divisions of the tetrachord are potentially infinite in number. It seems unlikely, however, that any great number of musically useful or theoretically interesting tetrachords has been omitted. Figures 9-1 through 9-6 show that the two-dimensional tetrachordal space is nearly filled by the tetrachords in the Catalog. The saturation of perceptual space is especially likely when one considers the finite resolving power of the ear, the limits on the accuracy and stability of analog and acoustic instruments, the quantizing errors of digital electronics, and our readiness to accept sufficiently close approximations to ideal tunings.

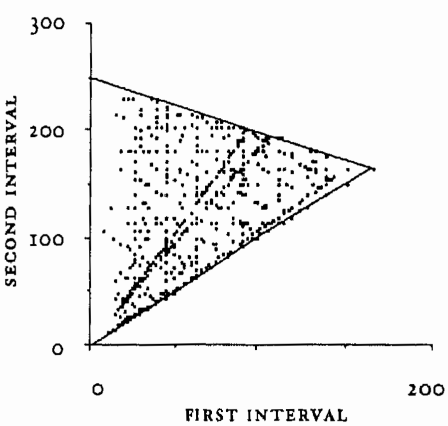
Nevertheless, processes such as searches through large microchromatic scales (chapter 7) and propriety calculations (chapter 5) will occasionally turn up new genera, so perhaps one should not be too complacent. The great majority of these new tetrachords, however, will resemble those already in the Catalog or be interchangeable with them for most melodic and harmonic purposes.

Organization of the Catalog

The tetrachords in the Main Catalog are listed by the size of their largest interval, which, in lieu of an historically validated term, has been called the



9-1. *Tetrachords in just intonation: smallest vs. largest intervals. Units in cents. The oblique lines are the upper and lower limits of the largest interval for each value of the smallest. This graph is limited to the tetrachords in the main, reduplicated, and miscellaneous lists.*



9-2. *Tetrachords in just intonation: first vs. second intervals. The oblique lines are the upper and lower limits of the second interval for each value of the first. This graph is limited to the tetrachords in the main, reduplicated, and miscellaneous lists.*

characteristic interval (CI). The term *apyknon* would have been used except that it has been traditionally employed to denote the sum of the two lower intervals of the diatonic genera. In diatonic tetrachords, this sum is greater than one half of the fourth.

Those tetrachords with CIs larger than 425 cents are classed as hyperenharmonic (after Wilson) and listed first. Next come the enharmonic with their *incomposite* CIs approximating major thirds. Chromatic and diatonic genera follow, the latter beginning when the CI falls below 250 cents.

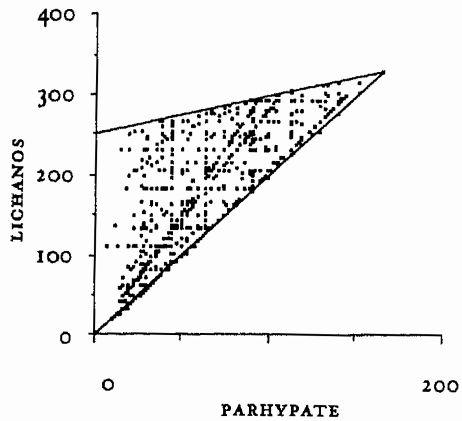
For each CI, the genera derived from the 1:1, 1:2, and 2:1 divisions of the pyknon or apyknon are listed first and followed by the other species of tetrachord with this CI. References to the earliest literature source and a brief discussion of the genus are given below each group.

In addition to the genera from the literature, the majority of the Main Catalog comprises tetrachords generated by the processes outlined in chapters 4 and 5. Both the 1:2 and 2:1 divisions are provided because both must be examined to select "strong," mostly superparticular forms in the Ptolemaic manner (chapter 2). If strict superparticularity is less important than convenience on the monochord or linear order, the 1:2 division is preferable, but recourse to the 2:1 may be necessary to discover the simplest form. For example, the threefold division of the 16/15 pyknon yields the notes 48 47 46 45. Ptolemy chose to recombine the first two intervals and reorder the third to obtain his enharmonic, 46/45 · 24/23 · 5/4.

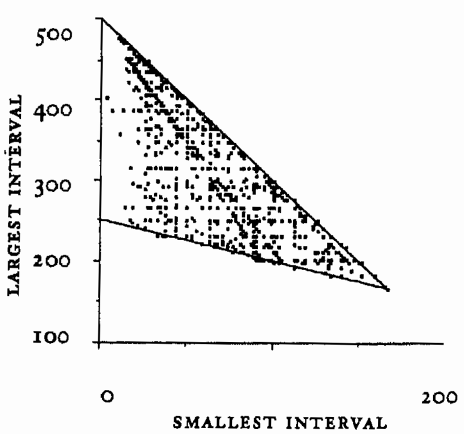
In general, only the simplest or mostly superparticular divisions are tabulated in this section; occasionally a theoretically interesting tetrachord without any near relatives will be found in the Miscellaneous list. Such isolated tetrachords are relatively uncommon. There are cases, however, in which all of the other divisions of a tetrachord's pyknon or apyknon have very complex ratios, and so closely resemble other tetrachords already tabulated that it did not seem fruitful to list them in a group under the CI in the Main Catalog.

"Miscellaneous" is a very elastic category. It consists of a collection of genera of diverse origin that I did not think interesting enough to list in the Main Catalog.

The order of intervals within each tetrachord is the canonical small, medium, and large in the case of the historical genera and their analogs. The new theoretical genera are generally listed in the order resulting from



9-3. *Tetrachords in just intonation: parhypatai vs. lichanoi. The oblique lines are the upper and lower limits of lichanos for each value of parhypate. This graph is limited to the tetrachords in the main, reduplicated, and miscellaneous lists.*



9-4. *Just and tempered tetrachords: smallest vs. largest intervals. The oblique lines are the upper and lower limits of the largest interval for each value of the smallest. This graph contains all the tetrachords in the Catalog.*

their generating process. It should be remembered, however, that all six permutations of the non-reduplicated genera and all three of the reduplicated are equally valid for musical experimentation.

With the exception of the Pythagorean 256/243 · 9/8 · 9/8 and Al-Farabi's 10/9 · 10/9 · 27/25, the genera with reduplicated intervals are given in the list of Reduplicated tetrachords.

Those tetrachords defined in either in "parts" of the tempered fourth or which consist solely of tempered intervals are to be found in the Tempered list. Needless to say, these tetrachords are a diverse lot, covering Aristoxenos's divisions, Greek Orthodox liturgical genera (in two systems — one of 28 parts to the fourth, the other of 30), and those derived from theoretical considerations. As some of the latter contain rational intervals as well, a separate list of Semi-tempered tetrachords is included.

No attempt has been made to catalog the very numerous tetrachords and tetrachord-like structures found in the non-zero modulo 12 equal temperaments of 4-17.

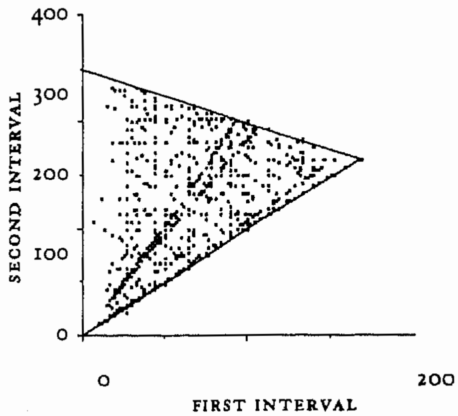
An index of sources for those tetrachords of historical provenance is provided.

Uniformity of sampling

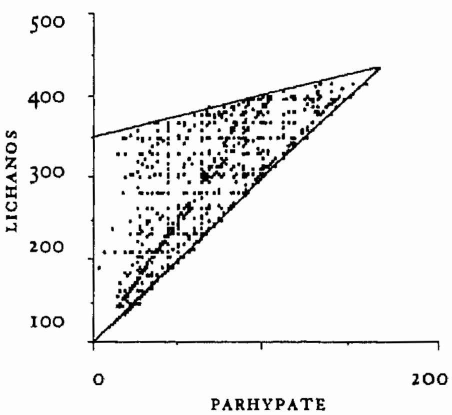
In order to show the uniformity with which the set of all possible tetrachords in just intonation has been sampled in the Catalogs of this chapter, the genera from the Main, Reduplicated, and Miscellaneous lists have been plotted in 9-1, 9-2 and 9-3. In 9-1, the smallest intervals are plotted against the largest intervals or CIs. As one may see, the area delineated by the two oblique lines is more or less uniformly filled. However, diagonal zones corresponding to genera with roughly equal and 1:2 divisions are evident. The tables are deliberately deficient in genera with commatic and sub-commatic intervals, as these are of little use melodically. The few examples in the tables are taken mostly from Hofmann's list of superparticular divisions (Vogel 1975) or generated by theoretical operations such as the means of chapter 4.

9-2 is a plot of the first versus the second intervals of the same tetrachords. Although the graph has a different shape, the same conclusions may be drawn.

9-3 is a third representation of the same data. In this case, cumulative rather than sequential intervals have been plotted. This mode reflects the Greek classification of tetrachords into primary genera (enharmonic,



9-5. *Just and tempered tetrachords: first vs. second intervals. The oblique lines are the upper and lower limits of the second interval for each value of the first. This graph contains all the tetrachords in the Catalog.*



9-6. *Just and tempered tetrachords: parhypatai vs. lichanoi. The oblique lines are the upper and lower limits of lichanos for each value of the parhypate. This graph contains all the tetrachords in the Catalog.*

chromatic and diatonic) and shades or nuances *(chroai)* of these genera. The primary distinction is based on the size of the uppermost interval, usually the CI except in Archytas's and Ptolemy's diatonics (28/27 · 8/7 · 9/8 and 16/15 · 9/8 · 10/9). The exact nuance or shade is then defined by the size of the first interval. The position of parhypate is equivalent to the size of the first interval and the position of lichanos is an inverse measure of the CI. This graph also reveals the relative uniformity of coverage and the excess of genera with 1:1 and 1:2 divisions.

The tetrachords in the Tempered and Semi-tempered lists were added to the set graphed in 9-1–3, and the entire collection replotted in 9-4–6. The largest empty spaces in the plots are thus filled. In a few cases, the gaps could be filled only by creating new genera specifically for this task. These have been marked in the Tempered tetrachord list.

The Main Catalog

HYPERENHARMONIC TETRACHORDS

H1. CHARACTERISTIC INTERVAL 13/10 454 CENTS

1 80/79 · 79/78 · 13/10 22 + 22 + 454

2 60/49 · 118/117 · 13/10 29 + 15 + 454

3 120/119 · 119/117 · 13/10 14 + 29 + 454

4 100/99 · 66/65 · 13/10 17 + 26 + 454 WILSON

The 13/10 would appear to be the upper limit for a genus-defining CI simply because the pyknotic intervals become too small to be melodically useful, however perceptible they might remain. In general, tetrachords with intervals less than 20 cents or with overly complex ratios will be relegated to the Miscellaneous listing at the end of the Catalog proper, unless there is some compelling reason, such as historical or literary reference, illustration of theory, or the like, to include them. The pyknon of this hyperenharmonic genus is the 40/39 (44 cents), which is very close to the Pythagorean double comma of $3^{24}/2^{38}$. Number 4 is from the unpublished notes of Ervin Wilson. See also Miscellaneous.

H2. CHARACTERISTIC INTERVAL 35/27 449 CENTS

5 72/71 · 71/70 · 35/27 24 + 25 + 449

6 108/107 · 107/105 · 35/27 16 + 33 + 449

7 54/53 · 106/105 · 35/27 32 + 16 + 449

8 64/63 · 81/80 · 35/27 27 + 22 + 449

This genus divides the 36/35 (49 cents), an interval found in Archytas's enharmonic and Avicenna's chromatic. Number 8 is found in Vogel's tuning for the Perfect Immutable System (Vogel 1963, 1967) and Erickson's (1965) analysis of Archytas's system (see chapter 6).

H3. Characteristic Interval 22/17 446 cents

9	68/67 · 67/66 · 22/17	26 + 26 + 446	
10	51/50 · 100/99 · 22/17	35 + 17 + 446	
11	102/101 · 101/99 · 22/17	17 + 35 + 446	
12	85/84 · 56/55 · 22/17	20 + 31 + 446	WILSON

The pyknon of this hyperenharmonic genus is 34/33 (52 cents), a quartertone. The intervening genera with pykna between 39/38 and 35/34 have not so far yielded melodically interesting, harmonically useful, nor mathematically elegant divisions, but see Miscellaneous for examples. This genus is replete with intervals of 17.

H4. Characteristic Interval 128/99 445 cents

13	66/65 · 65/64 · 128/99	26 + 27 + 445
14	99/98 · 49/48 · 128/99	18 + 36 + 445
15	99/97 · 97/96 · 128/99	35 + 18 + 445

The pyknon of this genus is 33/32 (53 cents), the octave-reduced thirty-third harmonic and an approximate quarter-tone.

H5. Characteristic Interval 31/24 443 cents

16	64/63 · 63/62 · 31/24	27 + 28 + 443
17	96/95 · 95/93 · 31/24	18 + 37 + 443
18	48/47 · 94/93 · 31/24	36 + 19 + 443

This hyperenharmonic genus divides the 32/31 (55 cents), an interval used in Didymos's enharmonic.

H6. Characteristic Interval 40/31 441 cents

19	62/61 · 61/60 · 40/31	28 + 29 + 441
20	93/92 · 46/45 · 40/31	19 + 38 + 441
21	93/91 · 91/90 · 40/31	38 + 19 + 441

The pyknon of this genus is 31/30 (57 cents), an interval which occurs in Didymos's enharmonic.

H7. Characteristic Interval 58/45 439 cents

22	60/59 · 59/58 · 58/45	29 + 30 + 439
23	90/89 · 89/87 · 58/45	19 + 39 + 439
24	45/44 · 88/87 · 58/45	39 + 20 + 439

25	120/119 · 119/116 · 58/45	14 + 44 + 439	

The pyknon of this hyperenharmonic genus is 30/29 (59 cents).

H8. Characteristic Interval 9/7 435 cents

26	56/55 · 55/54 · 9/7	31 + 32 + 435	WILSON
27	42/41 · 82/81 · 9/7	42 + 21 + 435	
28	84/83 · 83/81 · 9/7	21 + 42 + 435	
29	64/63 · 49/48 · 9/7	27 + 36 + 435	
30	70/69 · 46/45 · 9/7	25 + 38 + 435	
31	40/39 · 91/90 · 9/7	44 + 19 + 435	
32	112/111 · 37/36 · 9/7	16 + 47 + 435	
33	81/80 · 2240/2187 · 9/7	22 + 41 + 435	
34	9/7 · 119/117 · 52/51	435 + 29 + 34	

The pyknon of this prototypical hyperenharmonic genus (Wilson, unpublished) is Archytas's diesis, 28/27 (63 cents). Melodically, this genus bears the same relation to Aristoxenos's soft chromatic as Aristoxenos's enharmonic does to his syntonic (intense) chromatic. Number 26 is Wilson's original "hyperenharmonic" tetrachord. Divisions 29 and 31 are interesting in that their first intervals make, respectively, an 8/7 and a 15/13 with the subtonics hyperhypate (diatonic lichanos meson) and mese, and proslambanomenos and diatonic paranete diezeugmenon as well. Tetrachord number 32 is a good approximation to a hypothetical 1 + 3 + 26 parts, 17 + 50 + 433 cents—see also number 25 above. Number 33 occurs in Vogel's (1963, 1967) PIS tuning. Number 34 is a summation tetrachord from chapter 4.

H9. Characteristic Interval 104/81 433 cents

35	54/53 · 53/52 · 104/81	32 + 33 + 433
36	81/79 · 79/78 · 104/81	43 + 22 + 433
37	81/80 · 40/39 · 104/81	22 + 44 + 433

The pyknon of this genus is 27/26 (65 cents). This division is melodically similar to the 9/7 genus, though not harmonically. Number 37, when rearranged, generates a 15/13 with the subtonic.

H10. Characteristic Interval 50/39 430 cents

38	52/51 · 51/50 · 50/39	34 + 35 + 430
39	39/38 · 76/75 · 50/39	45 + 23 + 430
40	78/77 · 77/75 · 50/39	22 + 46 + 430

The pyknon is 26/25 (68 cents) and is inspired by Kathleen Schlesinger's (1939, 214) enharmonic Lydian harmonia.

Hii. Characteristic Interval 32/25 427 cents

41	50/49 · 49/48 · 32/25	35 + 36 + 427	
42	75/73 · 73/72 · 32/25	46 + 24 + 427	
43	75/74 · 37/36 · 32/25	23 + 47 + 427	

This genus divides the 25/24 minor semitone (71 cents). The 32/25 is the 3/2's complement of 75/64, the 5-limit augmented second (5/4 · 5/4 · 5/4 · 3/2, reduced to one octave).

ENHARMONIC TETRACHORDS

E1. Characteristic Interval 23/18 424 cents

44	48/47 · 47/46 · 23/18	36 + 37 + 424	SCHLESINGER
45	36/35 · 70/69 · 23/18	49 + 25 + 424	WILSON
46	72/71 · 71/69 · 23/18	24 + 50 + 424	
47	30/29 · 116/115 · 23/18	59 + 15 + 424	WILSON
48	60/59 · 118/115 · 23/18	29 + 45 + 424	

This genus divides the 24/23 (74 cents) and lies on the boundary between the enharmonic and hyperenharmonic genera. It is analogous to the 9/7 genus but divides the hemiolic chromatic rather than the soft or intense diesis. Numbers 45 and 47 are from Wilson. Number 44 (Schlesinger 1939, 214) is the lower tetrachord of her enharmonic Phrygian harmonia.

E2. Characteristic Interval 88/69 421 cents

49	46/45 · 45/44 · 88/69	38 + 39 + 421	
50	69/67 · 67/66 · 88/69	51 + 26 + 421	
51	69/68 · 34/33 · 88/69	25 + 52 + 421	

The pyknon of this enharmonic genus is 23/22 (77 cents).

E3. Characteristic Interval 50/41 421 cents

52	320/313 · 313/306 · 51/40	38 + 39 + 421	
53	480/473 · 473/459 · 51/40	25 + 52 + 421	
54	240/233 · 466/459 · 51/40	51 + 26 + 421	

The pyknon is 160/153 (77 cents). The 51/40 is the 3/2's complement of 20/17.

E4. Characteristic Interval 14/11 418 cents

55	44/43 · 43/42 · 14/11	40 + 41 + 418	
56	33/32 · 64/63 · 14/11	53 + 27 + 418	
57	66/65 · 65/63 · 14/11	26 + 54 + 418	
58	88/87 · 29/28 · 14/11	20 + 61 + 418	
59	36/35 · 55/54 · 14/11	49 + 32 + 418	

60 50/49 · 77/75 · 14/11 35 + 46 + 418

61 14/11 · 143/140 · 40/39 418 + 37 + 44

This is a new genus whose pyknon is 22/21 (81 cents). The 14/11 is a supramajor third found in the harmonic series between the fourteenth and eleventh partials. It occurs in the Partch diamond and other extended systems of just intonation.

E5. Characteristic Interval 80/63 414 cents

62 42/41 · 41/40 · 80/63 42 + 42 + 414

63 63/61 · 61/60 · 80/63 56 + 28 + 414

64 63/62 · 31/30 · 80/63 27 + 57 + 414

The pyknon of this enharmonic genus is 21/20 (84 cents), a common interval in septimal just intonation.

E6. Characteristic Interval 33/26 413 cents

65 208/203 · 203/198 · 33/26 42 + 43 + 413

66 312/307 · 307/297 · 33/26 28 + 57 + 413

67 312/302 · 302/297 · 33/26 56 + 29 + 413

68 52/51 · 34/33 · 33/26 34 + 52 + 413

69 26/25 · 100/99 · 33/26 68 + 18 + 413

70 78/77 · 28/27 · 33/26 22 + 63 + 413

The characteristic interval of this genus is the 3/2's complement of 13/11 and derives from the 22:26:33 triad. The pyknon is 104/99 (85 cents).

E7. Characteristic Interval 19/15 409 cents

71 40/39 · 39/38 · 19/15 44 + 45 + 409 ERATOSTHENES

72 30/29 · 58/57 · 19/15 59 + 30 + 409

73 60/59 · 59/57 · 19/15 29 + 60 + 409

74 28/27 · 135/133 · 19/15 63 + 26 + 409

The pyknon, 20/19 (89 cents), of this historically important genus is very close to the Pythagorean limma, 256/243. Number 71 is a good approximation to Aristoxenos's enharmonic of 3 + 3 + 24 "parts," and, in fact, is both Eratosthenes's enharmonic tuning and Ptolemy's misinterpretation of Aristoxenos's geometric scheme (Wallis 1682, 170). The next two entries are 2:1 and 1:2 divisions of the pyknon in analogy with the usual Ptolemaic and later Islamic practices. Number 73 is a hypothetical Ptolemaic interpretation of a (pseudo-)Aristoxenian 2 + 4 + 24 parts. An echo of this genus may appear as the sub-40 division found on the fingerboard of the Tanbur of Baghdad, a stringed instrument (Helmholtz [1877] 1954, 517).

The last species is an analog of Archytas's enharmonic and the first makes a 15/13 with the subtonic.

E8. Characteristic Interval 81/64 408 cents

75	512/499 · 499/486 · 81/64	45 + 46 + 408	BOETHIUS
76	384/371 · 742/729 · 81/64	60 + 31 + 408	
77	768/755 · 755/729 · 81/64	30 + 61 + 408	
78	40/39 · 416/405 · 81/64	44 + 46 + 408	
79	128/125 · 250/243 · 81/64	41 + 49 + 408	EULER
80	64/63 · 28/27 · 81/64	27 + 63 + 408	WILSON
81	$3^{24}/2^{38} \cdot 2^{46}/3^{29} \cdot 81/64$	47 + 43 + 408	
82	36/35 · 2240/2187 · 81/64	49 + 41 + 408	

In these tunings the limma, 256/243 (90 cents), has been divided. Number 75 is the enharmonic of Boethius and is obtained by a simple linear division of the pyknon. It represents Aristoxenos's enharmonic quite well, but see the preceding 19/15 genera for a solution more convenient on the monochord. In practice, the two (numbers 71 and 75) could not be distinguished by ear. Numbers 76 and 77 are triple divisions of the pyknon, for which Wilson's division is a convenient and harmonious approximation. Number 78 is an approximation to number 75, as is Euler's "old enharmonic" (Euler [1739] 1960, 170). Wilson's tuning (number 80) should also be compared to the Serre division of the 16/15 (5/4 genus). When number 80 is rearranged, the 28/27 will make a 7/6 with the subtonics hyperhypate or mese. In this form, it is a possible model for a tuning transitional between Aristoxenos's and Archytas's enharmonics. The purely Pythagorean division (number 81) is obtained by tuning five fifths down for the limma and twenty-four up for the double comma. Number 82 is found in Vogel's tuning (1963, 1967) and resembles Euler's (number 79).

E9. Characteristic Interval 24/19 404 cents

83	38/37 · 37/36 · 24/19	46 + 47 + 404	
84	57/55 · 55/54 · 24/19	62 + 32 + 404	
85	57/56 · 28/27 · 24/19	31 + 63 + 404	WILSON
86	76/75 · 25/24 · 24/19	23 + 71 + 404	
87	40/39 · 117/95 · 24/19	44 + 50 + 404	

The pyknon is 19/18 (94 cents). The interval of 24/19 derives from the 16:19:24 minor triad, which Shirlaw attributes to Ousley (Shirlaw 1917, 434) and which generates the corresponding tritriadic scale. It is the 3/2 complement of 19/16.

E10. CHARACTERISTIC INTERVAL 34/27 399 CENTS

88	36/35 · 35/34 · 34/27	49 + 50 + 399
89	27/26 · 52/51 · 34/27	65 + 34 + 399
90	54/53 · 53/51 · 34/27	32 + 67 + 399
91	24/23 · 69/68 · 34/27	74 + 25 + 399

This genus divides the 18/17 semitone of 99 cents, used by Vincenzo Galilei in his lute fretting (Barbour 1953; Lindley 1984). These genera are virtually equally-tempered and number 88 is an excellent approximation to Aristoxenos's enharmonic. It is also the first trichromatic of Schlesinger's Phrygian harmonia.

E11. CHARACTERISTIC INTERVAL 113/90 394 CENTS

92	240/233 · 233/226 · 113/90	51 + 53 + 394
93	180/173 · 346/339 · 113/90	69 + 35 + 394
94	360/353 · 353/339 · 113/90	34 + 70 + 394
95	30/29 · 116/113 · 113/90	59 + 45 + 394
96	40/39 · 117/113 · 113/90	44 + 60 + 394
97	60/59 · 118/113 · 113/90	29 + 75 + 394

These complex divisions derive from an attempt to interpret in Ptolemaic terms a hypothetical Aristoxenian genus of 7 + 23 parts. The inspiration came from Winnington-Ingram's 1932 article on Aristoxenos in which he discusses Archytas's 28/27 · 36/35 · 5/4 enharmonic genus and its absence from Aristoxenos's genera, despite the somewhat grudging acceptance of Archytas's other divisions. In Aristoxenian terms, Archytas's enharmonic would be 4 + 3 + 23 parts, and the first division is 3.5 + 3.5 + 23. Number 95 is the 4 + 3 division and 93 and 94 are 2:1 and 1:2 divisions of the complex pyknon of ratio 120/113 (104 cents). Numbers 96 and 97 are simplifications, while number 96 generates an ekbole of 5 dieses (15/13) with the subtonics hyperhypate and mese.

E12. CHARACTERISTIC INTERVAL 64/51 393 CENTS

98	34/33 · 33/32 · 64/51	52 + 53 + 393
99	51/50 · 25/24 · 64/51	34 + 71 + 393
100	49/48 · 51/49 · 64/51	36 + 69 + 393
101	68/65 · 65/64 · 64/51	78 + 27 + 393
102	68/67 · 67/64 · 64/51	26 + 79 + 393

The pyknon of this enharmonic genus is 17/16 (105 cents), the seventeenth harmonic and a basic interval in *septendecimal* just intonation.

E13. Characteristic Interval 5/4 386 cents

103	32/31 · 31/30 · 5/4	55 + 57 + 386	DIDYMOS
104	46/45 · 24/23 · 5/4	38 + 74 + 386	PTOLEMY
105	48/47 · 47/45 · 5/4	36 + 75 + 386	
106	28/27 · 36/35 · 5/4	63 + 49 + 386	ARCHYTAS
107	56/55 · 22/21 · 5/4	31 + 81 + 386	PTOLEMY?
108	40/39 · 26/25 · 5/4	44 + 68 + 386	AVICENNA
109	25/24 · 128/125 · 5/4	71 + 41 + 386	SALINAS
110	21/20 · 64/63 · 5/4	84 + 27 + 386	PACHYMERES
111	256/243 · 81/80 · 5/4	90 + 22 + 386	FOX-STRANGWAYS?
112	76/75 · 20/19 · 5/4	23 + 89 + 386	
113	96/95 · 19/18 · 5/4	18 + 94 + 386	WILSON
114	136/135 · 18/17 · 5/4	13 + 99 + 386	HOFMANN
115	256/255 · 17/16 · 5/4	7 + 105 + 386	HOFMANN
116	68/65 · 5/4 · 52/51	78 + 386 + 34	

These tunings are the most consonant of the shades of the enharmonic genera. Although Plato alludes to the enharmonic, the oldest tuning we actually have is that of Archytas (390 BCE). This tuning, number 106, clearly formed part of a larger musical system which included the subtonic and the tetrachord synemmenon as well as both the diatonic and chromatic genera (Winnington-Ingram 1932; Erickson 1965). Didymos's tuning is the 1:1 division of the 16/15 (112 cents) pyknon and dates from a time when the enharmonic had fallen out of use. Number 104 is undoubtedly Ptolemy's own, but the surviving manuscripts contain an extra page which lists number 107 instead. Wallis believed it to be a later addition, probably correctly. Numbers 104 and 105 are the 1:2 and 2:1 divisions, given as usual for illustrative and/or pedagogical purposes. The Avicenna tuning (D'Erlanger 1935, 154) has the 5/4 first in the original, following the usual practice of the Islamic theorists. In this form, it makes a 15/13 with the subtonic. Number 109 is Euler's enharmonic (Euler [1739] 1960, 178); Hawkins, however, attributes it to Salinas (Hawkins [1776] 1963, 27). Daniélou gives it in an approximation with 46/45 replacing the correct 128/125 (Daniélou 1943, 175). The Pachymeres enharmonic is attributed by Perrett to Tartini (Perrett 1926, 26), but Bryennios and Serre also list it.

Number 111 is given as *Rag Todi* by Fox-Strangways (1916, 121) and as *Gunakali* by Daniélou (1959, 134–135). The divisions with extraordinarily small intervals, numbers 114 and 115, were found by Hofmann in his computation of the 26 possible superparticular divisions of the 4/3 (Vogel 1975).

E14. Characteristic Interval 8192/6561 384 cents

117	4374/4235 · 4235/4096 · 8192/6561	57 + 57 + 384	
118	6561/6283 · 6283/6144 · 8192/6561	75 + 39 + 384	
119	6561/6422 · 3211/3072 · 8192/6561	37 + 77 + 384	
120	$3^{24}/2^{38} \cdot 2^{27}/3^{17} \cdot 8192/6561$	47 + 68 + 384	

The interval 8192/6561 is Helmholtz's *skhismic* major third, which is generated by tuning eight fifths down and five octaves up (Helmholtz [1877] 1954, 432). The pyknon is the apotome, 2187/2048 (114 cents). It has been linearly divided in the first three tetrachords above, but a purely Pythagorean division is given as number 120.

E15. Characteristic Interval 56/45 379 cents

121	30/29 · 29/28 · 56/45	59 + 60 + 379	PTOLEMY
122	45/43 · 43/42 · 56/45	79 + 41 + 379	
123	45/44 · 22/21 · 56/45	39 + 53 + 379	
124	25/24 · 36/35 · 56/45	71 + 49 + 379	
125	80/77 · 33/32 · 56/45	66 + 53 + 379	
126	60/59 · 59/56 · 56/45	29 + 90 + 379	
127	40/39 · 117/112 · 56/45	44 + 76 + 379	
128	26/25 · 375/364 · 56/45	68 + 52 + 379	

The pyknon is 15/14 (119 cents). Number 121 is Ptolemy's interpretation of Aristoxenos's soft chromatic, 4 + 4 + 22 parts. Number 125 is a Ptolemaic interpretation of a hypothetical 4.5 + 3.5 + 22 parts, an approximation to Archytas's enharmonic (Winnington-Ingram 1932). Number 124 is a simplification of the former tuning, and numbers 122 and 123 are the familiar threefold divisions. Number 128 is a summation tetrachord.

E16. Characteristic Interval 41/33 376 cents

129	88/85 · 85/82 · 41/33	60 + 62 + 376
130	42/41 · 22/21 · 41/33	42 + 81 + 376
131	44/43 · 43/41 · 41/43	39 + 82 + 376

This genus is an attempt to approximate a theoretical genus, 62.5 + 62.5 + 375 cents, which would lie on the border between the chromatic and enharmonic genera. Number 129 is quite close, and numbers 130 and 131 are 1:2 and 2:1 divisions of the complex 44/41 (122 cents) pyknon.

C1. Characteristic Interval 36/29 374 cents

132	29/28 · 28/27 · 36/29	61 + 63 + 374	
133	87/85 · 85/81 · 36/29	40 + 83 + 374	
134	87/83 · 83/81 · 36/29	81 + 42 + 374	

This genus is also an approximation to 62.5 + 62.5 + 375 cents. The 36/29 is from the 24:29:36 triad and tritriadic scale. The pyknon is 29/27 (124 cents).

C2. Characteristic Interval 26/21 370 cents

135	28/27 · 27/26 · 26/21	63 + 65 + 370	SCHLESINGER
136	21/20 · 40/39 · 26/21	85 + 44 + 370	
137	42/41 · 41/39 · 26/21	42 + 87 + 370	
138	24/23 · 161/156 · 26/21	74 + 55 + 370	

This genus divides the pyknon, 14/13 (128 cents) and approximates Aristoxenos's soft chromatic. Number 135 is from Schlesinger (1933) and is a first tetrachord of a modified Mixolydian harmonia.

C3. Characteristic Interval 21/17 366 cents

139	136/131 · 131/126 · 21/17	65 + 67 + 366	
140	102/97 · 194/189 · 21/17	87 + 45 + 366	
141	204/199 · 199/189 · 21/17	43 + 89 + 366	
142	64/63 · 17/16 · 21/17	27 + 105 + 366	
143	34/33 · 22/21 · 21/17	52 + 81 + 366	
144	40/39 · 221/210 · 21/17	44 + 88 + 366	
145	24/23 · 391/378 · 21/17	74 + 59 + 366	
146	28/27 · 51/49 · 21/17	63 + 69 + 366	

The pyknon is 68/63 (132 cents). Number 139 is a very close approximation of Aristoxenos's soft chromatic, 4 + 4 + 22 "parts," as is number 146 also. Numbers 144 and 146 make intervals of 15/13 and 7/6, respectively, with their subtonics.

C4. Characteristic Interval 100/81 365 cents

147	27/26 · 26/25 · 100/81	65 + 68 + 365	
148	81/77 · 77/75 · 100/81	87 + 46 + 365	
149	81/79 · 79/75 · 100/81	45 + 88 + 365	
150	81/80 · 16/15 · 100/81	22 + 112 + 365	
151	51/50 · 18/17 · 100/81	34 + 99 + 365	
152	36/35 · 21/20 · 100/81	49 + 85 + 365	

153	40/39 · 1053/1000 · 100/81	44 + 89 + 365	
154	135/128 · 128/125 · 100/81	92 + 41 + 365	DANIÉLOU
155	24/23 · 207/200 · 100/81	74 + 60 + 365	

The pyknon is the great limma or large chromatic semitone, 27/25 (133 cents). Daniélou listed his tetrachord in approximate form with 46/45 instead of the correct 128/125. (Daniélou 1943, 175). Number 147 is a close approximation to Aristoxenos's soft chromatic, but the rest of the divisions are rather complex.

C5. Characteristic Interval 37/30 363 cents

156	80/77 · 77/74 · 37/30	66 + 69 + 363	PTOLEMY
157	20/19 · 38/37 · 37/30	89 + 46 + 363	
158	40/39 · 39/37 · 37/30	44 + 91 + 363	
159	30/29 · 116/111 · 37/30	59 + 76 + 363	
160	60/59 · 118/111 · 37/30	29 + 106 + 363	

This complex chromatic genus divides the 40/37 (135 cents). Number 156 is Ptolemy's linear interpretation of Aristoxenos's hemiolic chromatic, 4.5 + 4.5 + 21 "parts," with its characteristic neutral third and 3/4-tone pyknon. This division closely approximates his soft chromatic, indicating that Ptolemy's interpretation in terms of the aliquot parts of a real string was erroneous and that Aristoxenos really did mean something conceptually similar to equal temperament. However, Ptolemy's approach and the resulting tetrachords are often interesting in their own right. For example, number 157 could be considered as a Ptolemaic version of Aristoxenos's 1/2 + 1/4 + 1 3/4 tones, 6 + 3 + 21 "parts," a genus rejected as unmelodic because the second interval is smaller than the first (Winnington-Ingram 1932). The remaining genera are experimental.

C6. Characteristic Interval 16/13 359 cents

161	26/25 · 25/24 · 16/13	68 + 71 + 359
162	39/37 · 37/36 · 16/13	91 + 47 + 359
163	39/38 · 19/18 · 16/13	45 + 94 + 359
164	65/64 · 16/15 · 16/13	27 + 112 + 359
165	52/51 · 17/16 · 16/13	34 + 105 + 359
166	40/39 · 169/160 · 16/13	44 + 95 + 359
167	28/27 · 117/112 · 16/13	63 + 76 + 359
168	169/168 · 14/13 · 16/13	11 + 128 + 359
169	22/21 · 91/88 · 16/13	81 + 58 + 359

The pyknon of this genus, which lies between the soft and hemiolic chromatics of Aristoxenos, is 13/12 (139 cents). Number 169 is a summation tetrachord from chapter 4.

C7. Characteristic Interval 27/22 355 cents

170	176/169 · 169/162 · 27/22	70 + 73 + 355	
171	132/125 · 250/243 · 27/22	94 + 49 + 355	
172	264/257 · 257/243 · 27/22	47 + 97 + 355	
173	28/27 · 22/21 · 27/22	63 + 81 + 355	
174	55/54 · 16/15 · 27/22	32 + 112 + 355	
175	40/39 · 143/135 · 27/22	44 + 100 + 355	

The *Wosta of Zalzal*, a neutral third of 355 cents, is exploited in this hemiolic chromatic genus whose pyknon is 88/81 (143 cents), an interval found in certain Islamic scales (D'Erlanger 1935).

C8. Characteristic Interval 11/9 347 cents

176	24/23 · 23/22 · 11/9	74 + 77 + 347	WINNINGTON-INGRAM
177	18/17 · 34/33 · 11/9	99 + 52 + 347	
178	36/35 · 35/33 · 11/9	49 + 102 + 347	
179	45/44 · 16/15 · 11/9	39 + 112 + 347	
180	56/55 · 15/14 · 11/9	31 + 119 + 347	
181	78/77 · 14/13 · 11/9	22 + 128 + 347	
182	20/19 · 57/55 · 11/9	89 + 62 + 347	
183	30/29 · 58/55 · 11/9	59 + 92 + 347	
184	28/27 · 81/77 · 11/9	63 + 88 + 347	
185	40/39 · 117/110 · 11/9	44 + 107 + 347	

This genus is the simplest realization of Aristoxenos's hemiolic chromatic. Winnington-Ingram mentions number 176 in his 1932 article on Aristoxenos but rejects it, despite using 12/11 · 11/9 to construct his spondeion scale in an earlier paper (Winnington-Ingram 1928). In view of the widespread use of 3/4-tone and neutral third intervals in extant Islamic music and the use of 12/11 by Ptolemy in his intense chromatic and equable diatonic genera, I see no problems with accepting Aristoxenos's genus, 4.5 + 4.5 + 21 "parts," as recording an actual tuning, traces of which are still to be found in the Near East. Ptolemy, it should be remembered, claimed that the intense chromatic, 22/21 · 12/11 · 7/6, was used in popular lyra and kithara tunings (Wallis 1682, 84, 178, 208) and that his equable diatonic sounded rather foreign and rustic. Schlesinger identifies it with the first tetrachord of her chromatic Phrygian harmonia (Schlesinger 1933; Schlesinger 1939, 214). The pyknon of this chromatic genus is 12/11 (151 cents). Number 176 may be written as 5 + 5 + 20 Ptolemaic "parts" (120 115 110 90), rather than the 4.5 + 4.5 + 21 of Aristoxenian theory. A number of other divisions are shown, including the usual 1:2 and 2:1, as well as the neo-Archytan 28/27 and 40/39 types.

C9. Characteristic Interval 39/32 342 cents

186	256/245 · 245/234 · 39/32	76 + 80 + 342	
187	384/373 · 373/351 · 39/32	50 + 105 + 342	
188	192/181 · 362/351 · 39/32	102 + 53 + 342	
189	64/63 · 14/13 · 39/32	27 + 128 + 342	

This genus employs the 3/2's complement of 16/13, the tridecimal neutral third, found in the 26:32:39 triad. The unusually complex pyknon is 128/117 (156 cents).

C10. Characteristic Interval 28/23 341 cents

190	23/22 · 22/21 · 28/23	76 + 81 + 341	WILSON
191	69/65 · 65/63 · 28/23	103 + 54 + 341	
192	69/67 · 67/63 · 28/23	51 + 107 + 341	
193	46/45 · 15/14 · 28/23	38 + 119 + 341	

This neutral third genus is from Wilson. The pyknon is 23/21 (157 cents).

C11. Characteristic Interval 17/14 336 cents

194	112/107 · 107/102 · 17/14	79 + 83 + 336	
195	168/158 · 158/153 · 17/14	106 + 56 + 336	
196	168/163 · 163/153 · 17/14	52 + 110 + 336	
197	52/51 · 14/13 · 17/14	34 + 128 + 336	
198	28/27 · 18/17 · 17/14	63 + 99 + 336	
199	35/34 · 16/15 · 17/14	50 + 112 + 336	
200	40/39 · 91/85 · 17/14	44 + 118 + 336	
201	17/14 · 56/55 · 55/51	336 + 31 + 131	
202	17/14 · 56/53 · 53/51	336 + 95 + 67	

This chromatic genus uses Ellis's supraminor third, 17/14 (Helmholtz [1877] 1954, 455), which occurs in his septendecimal interpretation of the diminished seventh chord, 10:12:14:17. The pyknon is 56/51 (162 cents).

C12. Characteristic Interval 40/33 333 cents

203	22/21 · 21/20 · 40/33	81 + 85 + 333	
204	33/32 · 31/30 · 40/33	108 + 57 + 333	
205	33/32 · 16/15 · 40/33	53 + 112 + 333	
206	55/54 · 27/25 · 40/33	32 + 133 + 333	

207	66/65 · 13/12 · 40/33	26 + 139 + 333	
208	18/17 · 187/180 · 40/33	99 + 66 + 333	

The pyknon of this genus is 11/10 (165 cents), an interval which appears in Ptolemy's equable diatonic and elsewhere. Number 208 is a summation tetrachord from chapter 4.

C13. CHARACTERISTIC INTERVAL 29/24 328 CENTS

209	64/61 · 61/58 · 29/24	83 + 87 + 328	
210	16/15 · 30/29 · 29/24	112 + 59 + 328	SCHLESINGER
211	32/31 · 31/29 · 29/24	55 + 115 + 328	SCHLESINGER

The interval 29/24 is found in some of Schlesinger's harmoniai when she tries to correlate her theory of linearly divided octaves with Greek notation (Schlesinger 1939, 527–8). The results agree neither with the commonly accepted interpretation of the notation, nor with the canonical forms of the harmoniai given elsewhere in her book. The 29/24 is also part of the 24:29:36 triad and its 3/2's complement generates the 36/29 genus. The pyknon is 32/29 (170 cents).

C14. CHARACTERISTIC INTERVAL 6/5 316 CENTS

212	20/19 · 19/18 · 6/5	89 + 94 + 316	ERATOSTHENES
213	28/27 · 15/14 · 6/5	63 + 119 + 316	PTOLEMY
214	30/29 · 29/27 · 6/5	59 + 123 + 316	
215	16/15 · 25/24 · 6/5	112 + 71 + 316	DIDYMOS
216	40/39 · 13/12 · 6/5	44 + 139 + 316	BARBOUR
217	55/54 · 12/11 · 6/5	32 + 151 + 316	BARBOUR
218	65/63 · 14/13 · 6/5	54 + 128 + 316	
219	22/21 · 35/33 · 6/5	81 + 102 + 316	
220	21/20 · 200/189 · 6/5	85 + 97 + 316	PERRETT
221	256/243 · 6/5 · 135/128	90 + 316 + 92	XENAKIS
222	60/59 · 59/54 · 6/5	29 + 153 + 316	
223	80/77 · 77/72 · 6/5	66 + 116 + 316	
224	24/23 · 115/108 · 6/5	74 + 109 + 316	
225	88/81 · 45/44 · 6/5	143 + 39 + 316	
226	46/45 · 6/5 · 25/23	38 + 316 + 144	
227	52/51 · 85/78 · 6/5	34 + 149 + 316	WILSON
228	100/99 · 11/10 · 6/5	17 + 165 + 316	HOFMANN
229	34/33 · 6/5 · 55/51	52 + 316 + 131	
230	6/5 · 35/32 · 64/63	316 + 155 + 27	
231	6/5 · 2240/2187 · 243/224	316 + 41 + 141	

This genus is the most consonant of the chromatic genera. Number 212 is the chromatic of Eratosthenes and is identical to Ptolemy's interpretation of Aristoxenos's intense chromatic genus. It is likely, however, that Aristoxenos's genus corresponds to one of the 32/27 genera. Number 213 is Ptolemy's soft chromatic and is the 2:1 division reordered. Number 214 is the 1:2 division and a Ptolemaic interpretation of a 4 + 8 + 18 "parts." Didymos's tuning is probably the most consonant, although it violates the usual melodic canon of Greek theory that the smallest interval must be at the bottom of the tetrachord. In reverse order, this tuning is produced by the seventh of Proclus's ten means (Heath 1921). Archytas's enharmonic and diatonic tunings also violate this rule; the rule may either be later or an ideal theoretical principle. Numbers 216 and 217 are from Barbour (1951, 23). Perrett's tetrachord, like one of the 25/21 genera, is found to occur unexpectedly in his new scale (Perrett 1926, 79). The Xenakis tetrachord (number 221) is from the article, "Towards a Metamusic," which has appeared in different translations in different places (Xenakis 1971). It also appears in Archytas's system according to Erickson (1965). The Hofmann genus is from Vogel (1975). Numbers 230 and 231 are found in Vogel's tuning (1963, 1967) and chapter 6. The pyknon is the minor tone 10/9 (182 cents).

C15. Characteristic Interval 25/21 302 cents

232	56/53 · 53/50 · 25/21	97 + 99 + 302	
233	14/13 · 26/25 · 25/21	128 + 68 + 302	
234	28/27 · 27/25 · 25/21	63 + 133 + 302	
235	21/20 · 16/15 · 25/21	84 + 112 + 302	PERRETT
236	40/39 · 273/250 · 25/21	44 + 152 + 302	

This genus whose pyknon is 28/25 (196 cents) is inspired by number 235, a tetrachord from Perrett (1926, 80). Number 232 is virtually equally tempered and number 234 is an excellent approximation to Aristoxenos's 1/3 + 2/3 + 1 1/2 tones, 4 + 8 + 18 "parts."

C16. Characteristic Interval 19/16 298 cents

237	128/121 · 121/114 · 19/16	97 + 103 + 298	
238	96/89 · 178/171 · 19/16	131 + 69 + 298	
239	192/185 · 185/171 · 19/16	64 + 136 + 298	
240	20/19 · 19/16 · 16/15	89 + 298 + 112	KORNERUP
241	256/243 · 81/76 · 19/16	90 + 110 + 298	BOETHIUS
242	96/95 · 10/9 · 19/16	18 + 182 + 298	WILSON

243	64/63 · 21/19 · 19/16	27 + 173 + 298	
244	40/39 · 104/95 · 19/16	44 + 157 + 298	

The characteristic ratio for this genus derives from the 16:19:24 minor triad (see the 24/19 genus). The pyknon is the complex interval 64/57 (201 cents). Number 241 is from Boethius (1838, 6). The Kornerup tetrachord (1934, 10) also corresponds to a Ptolemaic interpretation of one of Athanasopoulos's (1950) Byzantine tunings, 6 + 18 + 6 "parts." As 19/16 · 20/19 · 16/15, it is one of the "mean" tetrachords.

C17. CHARACTERISTIC INTERVAL 32/27 294 CENTS

245	18/17 · 17/16 · 32/27	99 + 105 + 294	ARISTIDES QUINT.
246	27/25 · 25/24 · 32/27	133 + 71 + 294	
247	27/26 · 13/12 · 32/27	65 + 139 + 294	BARBOUR?
248	28/27 · 243/224 · 32/27	63 + 141 + 294	ARCHYTAS
249	256/243 · 2187/2048 · 32/27	90 +114 + 294	GAUDENTIUS
250	81/80 · 10/9 · 32/27	22 + 182 + 294	BARBOUR?
251	33/32 · 12/11 · 32/27	53 + 151 + 294	BARBOUR?
252	45/44 + 11/10 · 32/27	39 + 165 + 294	BARBOUR?
253	21/20 · 15/14 · 32/27	84 + 119 + 294	PERRETT
254	135/128 · 16/15 · 32/27	92 + 112 + 294	
255	36/35 · 35/32 · 32/27	49 + 155 + 294	WILSON
256	49/48 · 54/49 · 32/27	36 + 168 + 294	WILSON
257	243/230 · 230/216 · 32/27	95 + 109 + 294	PS.-PHILOLAUS?
258	243/229 · 229/216 · 32/27	103 +101+ 294	
259	20/19 · 171/160 · 32/27	89 + 115 + 294	
260	23/22 · 99/92 · 32/27	77 + 127 + 294	
261	24/23 · 69/64 · 32/27	74 + 130 + 294	
262	40/39 + 351/320 · 32/27	44 + 160 + 294	
263	14/13 · 117/112 · 32/27	128 + 76 + 294	

These chromatic genera are derived from the traditional "Pythagorean" tuning (perfect fourths, fifths, and octaves), which is actually of Sumero-Babylonian origin (Duchesne-Guillemin 1963, 1969; Kilmer 1960), by changing the pitch of the second string, the parhypate or trite. Number 245, the 1:1 division of the 9/8 pyknon (204 cents), is from from the late classical writer, Aristides Quintilianus (Meibomius 1652, 123). Tunings numbers 246 and 254 are of obscure origin. They were constructed after reading a passage in Hawkins ([1776] 1963, 37) which quotes Wallis as crediting Mersenne with the discovery of the 27/25 and 135/128 semitones and their 9/8 complements. However, the discussion is about diatonic genera, not chromatic, and it is unclear to me whether Mersenne really did construct these two chromatic tetrachords. Archytas's chromatic, number 248, has been identified with Aristoxenos's 1/3 + 2/3 + 1 1/2 tones by Winnington-Ingram (1932) and number 247 is a good approximation to his 1/2 + 1/2 + 1 1/2 tones. Number 249 is the unaltered Pythagorean version from Gaudentius. The Barbour tetrachords derive from his discussion of different superparticular divisions of the 9/8 (Barbour 1951, 154–156). Although tetrachords are mentioned, it is not clear that he ever actually constructed these divisions. Perrett discovered number 253, like number 235 above, in his scale after it was constructed. Both Chaignet (1874, 231) and McClain (1978, 160) quote (*Ps.*)-Philolaus as dividing the tone into 27 parts, 13 of which go to the minor semitone, and 14 to the major. Number 257 is the result of this division and number 258 has the parts taken in reverse order. It would seem that number 245 and number 258 are essentially equivalent to Aristoxenos's theoretical intense chromatic and that numbers 254, 257, 259, and probably 253 as well, are equivalent to Gaudentius's Pythagorean tuning. The presence of secondary ratios of 5 and 7 in number 253 and number 254 suggests that the equivalences would be melodic rather than harmonic. The last tuning is a summation tetrachord from chapter 4.

C18. Characteristic Interval 45/38 293 cents

264	304/287 · 287/270 · 45/38	100 + 106 + 293
265	456/439 · 439/405 · 45/38	66 + 140 + 293
266	228/211 · 422/405 · 45/38	134 + 71 + 293
267	19/18 · 16/15 · 45/38	94 + 112 + 293
268	76/75 · 10/9 · 45/38	23 + 182 + 293
269	38/35 · 28/27 · 45/38	142 + 63 + 293

This genus uses the 45/38, the 3/2's complement of 19/15. The pyknon is 152/135 (205 cents). Number 264 is a reasonable approximation to the intense chromatic and number 269 is similar to Archytas's chromatic, if rearranged with the 28/27 first.

C19. Characteristic Interval 13/11 289 cents

270	88/83 · 83/78 · 13/11	101 + 108 + 289
271	66/61 · 122/117 · 13/11	136 + 72 + 289
272	132/127 · 127/117 · 13/11	67 + 142 + 289
273	14/13 · 22/21 · 13/11	128 + 81 + 289
274	40/39 · 11/10 · 13/11	44 + 165 + 289

275	66/65 · 10/9 · 13/11	26 + 182 + 289	WILSON
276	27/26 · 88/81 · 13/11	65 + 143 + 289	
277	28/27 · 99/91 · 13/11	63 + 146 + 289	

This experimental genus divides a pyknon of 44/39 (209 cents), an interval also appearing in William Lyman Young's diatonic lyre tuning (Young 1961). The 13/11 is a minor third which appears in 13-limit tunings and with its 3/2's complement, 33/26, generates the 22:26:33 tritriadic scale.

C20. Characteristic Interval 33/28 284 cents

278	224/211 · 211/198 · 33/28	104 + 110 + 284
279	336/323 · 323/297 · 33/28	68 + 145 + 284
280	168/155 · 310/297 · 33/28	139 + 74 + 284
281	56/55 · 10/9 · 33/28	31 + 182 + 284
282	16/15 · 35/32 · 33/28	112 + 102 + 284
283	34/33 · 33/28 · 56/51	52 + 284 + 162

The characteristic interval of this genus is the 3/2's complement of 14/11, 33/28. The pyknon is 112/99 (214 cents).

C21. Characteristic Interval 20/17 281 cents

284	17/16 · 16/15 · 20/17	105 + 112 + 281
285	51/47 · 47/45 · 20/17	142 + 75 + 281
286	51/49 · 49/45 · 20/17	69 + 147 + 281
287	34/33 · 11/10 · 20/17	52 + 165 + 281
288	51/50 · 10/9 · 20/17	34 + 182 + 281
289	40/39 · 221/200 · 20/17	44 + 173 + 281
290	28/27 · 153/140 · 20/17	63 + 154 + 281
291	21/20 · 20/17 · 68/63	85 + 281 + 132
292	68/65 · 13/12 · 20/17	78 + 139 + 281
293	34/31 · 31/30 · 20/17	160 + 57 + 281
294	68/61 · 61/60 · 20/17	188 + 29 + 281
295	68/67 · 67/57 · 19/17	26 + 280 + 193
296	68/67 · 67/60 · 20/17	26 + 191 + 281

The pyknon is 17/15 (217 cents). Intervals of 17 are becoming increasingly common in justly-intoned music. This would appear to be a metaphysical phenomenon of considerable philosophical interest (Polansky, personal communication).

C22. Characteristic Interval 27/23 278 cents

297	184/173 · 173/162 · 27/23	107 + 114 + 278
298	276/265 · 265/243 · 27/23	70 + 150 + 278

299	138/127 · 254/243 · 27/2	144 + 77 + 278	
300	28/27 · 23/21 · 27/23	63 + 157 + 278	
301	23/22 · 88/81 · 27/23	77 + 143 + 278	
302	46/45 · 10/9 · 27/23	38 + 182 + 278	

This genus exploits the 3/2's complement of 23/18, which is derived from the 18:23:27 triad. The pyknon is 92/81 (220 cents).

C23. Characteristic Interval 75/64 275 cents

303	512/481 · 481/450 · 75/64	108 + 115 + 275	
304	768/737 · 737/675 · 75/64	71 + 152 + 275	
305	384/353 · 706/675 · 75/64	146 + 78 + 275	
306	16/15 · 75/64 · 16/15	112 + 275 + 112	HELMHOLTZ

The pyknon is 256/225 (223 cents). The 75/64 is the 5-limit augmented second, which appears, for example, in the harmonic minor scale. Helmholtz's tetrachord is from (Helmholtz [1877] 1954, 263).

C24. Characteristic Interval 7/6 267 cents

307	16/15 · 15/14 · 7/6	112 + 119 + 267	AL-FARABI
308	22/21 · 12/11 · 7/6	81 + 151 + 267	PTOLEMY
309	24/23 · 23/21 · 7/6	74 + 157 + 267	
310	20/19 · 38/35 · 7/6	89 + 142 + 267	PTOLEMY
311	10/9 · 36/35 · 7/6	182 + 49 + 267	AVICENNA
312	64/63 · 9/8 · 7/6	27 + 204 + 267	BARBOUR
313	92/91 · 26/23 · 7/6	19 + 212 + 267	
314	256/243 · 243/224 · 7/6	90 + 141 + 267	HIPKINS
315	40/39 · 39/35 · 7/6	44 + 187 + 267	
316	18/17 · 7/6 · 68/63	99 + 267 + 132	
317	50/49 · 7/6 · 28/25	35 + 267 + 196	
318	14/13 · 7/6 · 52/49	128 + 267 + 103	
319	46/45 · 180/161 · 7/6	38 + 193 + 267	
320	28/27 · 54/49 · 7/6	63 + 168 + 267	
321	120/113 · 113/105 · 7/6	104 + 127 + 267	
322	60/59 · 118/105 · 7/6	29 + 202 + 267	
323	30/29 · 116/105 · 7/6	59 + 172 + 267	
324	88/81 · 81/77 · 7/6	143 + 88 + 267	
325	120/119 · 17/15 · 7/6	14 + 217 + 267	
326	27/25 · 7/6 · 200/189	133 + 267 + 98	
327	26/25 · 7/6 · 100/91	68 + 267 + 163	

328 7/6 · 1024/945 · 135/128 267 + 139 + 92

The pyknon of this intense chromatic is the septimal tone, 8/7 (231 cents). Number 307 is given by Al-Farabi (D'Erlanger 1930, 104) and by Sachs (1943, 282) in rearranged form as the lower tetrachord of the modern Islamic mode, *Higaz*. The Turkish mode, *Zirgule*, has also been reported to contain this tetrachord, also with the 7/6 medially (Palmer 1967?). Vincent attributes this division to the Byzantine theorist, Pachymeres (Vincent 1847). This tuning is also produced by the harmonic mean operation. Ptolemy's first division (number 308) is his intense chromatic (Wallis 1682, 172), and his second (number 310) is his interpretation of Aristoxenos's soft diatonic, 6 + 9 + 15 "parts". In this instance, Ptolemy is not too far from the canonical 100 + 150 + 250 cents, though Hipkins's semi-Pythagorean solution (number 314) is more realistic (Vogel 1963). His tuning is also present in Erickson's (1965) interpretation of Archytas's system. The Avicenna tetrachord, number 311, (D'Erlanger 1935, 152) sounds, surprisingly, rather diatonic. Barbour's (1951, 23–24) tuning (number 312) is particularly attractive when arranged as 9/8 · 64/63 · 7/6. It also generates the 16:21:24 tritriadic and its conjugate. Vogel (1975, 207) lists it also. Number 328 is found in Vogel's tuning (chapter 6 and Vogel 1963, 1967). The remaining divisions are new tetrachords intended as variations on the soft diatonic-intense chromatic genus or as approximations of various Byzantine tetrachords as described by several authors (Xenakis 1971; Savas 1965; Athanasopoulos 1950).

C25. Characteristic Interval 136/117 261 cents

329	78/73 · 73/68 · 136/117	115 + 123 + 261
330	117/112 · 56/51 · 136/117	76 + 162 + 261
331	117/107 · 107/102 · 136/117	155 + 83 + 261
332	52/51 · 9/8 · 136/117	34 + 204 + 261

The pyknon of this complex genus is 39/34 (238 cents). Number 332 generates the 26:34:39 tritriadic.

C26. Characteristic Interval 36/31 259 cents

333	31/29 · 29/27 · 36/31	115 + 124 + 259
334	93/89 · 89/81 · 36/31	76 + 163 + 259
335	93/85 · 85/81 + 36/31	156 + 83 + 259

The pyknon is 31/27 (239 cents). The 36/31 is the 3/2's complement of 31/24, which defines a hyperenharmonic genus.

C27. Characteristic Interval 80/69 256 cents

336	46/43 · 43/40 · 80/69	117 + 125 + 256	
337	23/21 · 21/20 · 80/69	157 + 85 + 256	
338	23/22 · 11/10 · 80/69	77 + 165 + 256	
339	46/45 · 9/8 · 80/69	38 + 204 + 256	

The genus derives from number 339 which generates the 20:23:30 and 46:60:69 tritriadics. The pyknon is 23/20 (242 cents). This and the next few genera are realizations of Aristoxenos's soft diatonic.

C28. Characteristic Interval 22/19 254 cents

340	76/71 · 71/66 · 22/19	118 + 126 + 254	
341	57/52 · 104/99 · 22/19	159 + 85 + 254	
342	114/109 · 109/99 · 22/19	78 + 167 + 254	
343	19/18 · 12/11 · 22/19	94 + 151 + 254	SCHLESINGER
344.	34/33 · 19/17 · 22/19	52 + 192 + 254	
345	40/39 · 247/220 · 22/19	44 + 200 + 254	

This genus is a good approximation to the soft diatonic. Number 343 is from a folk scale (Schlesinger 1939, 297). Tetrachord numbers 344 and 345 are close to 3 + 12 + 15 "parts", a neo-Aristoxenian genus which mixes enharmonic and diatonic intervals. The pyknon is 38/33 (244 cents).

C29. Characteristic Interval 52/45 250 cents

346	15/14 · 14/13 · 52/45	119 + 128 + 250
347	45/41 · 41/39 · 52/45	161 + 87 + 250
348	45/43 · 43/39 · 52/45	78 + 169 + 250
349	24/23 · 115/104 · 52/45	74 + 174 + 250
350	40/39 · 9/8 · 52/45	44 + 204 + 250
351	18/17 · 85/78 · 52/45	99 + 149 + 250
352	45/44 · 44/39 · 52/45	39 + 209 + 250
353	65/63 · 28/25 · 52/45	54 + 196 + 250
354	55/52 · 12/11 · 52/45	97 + 151 + 250
355	60/59 · 59/45 · 52/45	29 + 219 + 250
356	20/19 · 52/45 · 57/52	89 + 250 + 149
357	27/26 · 10/9 · 52/45	66 + 182 + 250
358	11/10 · 150/143 · 52/45	165 + 83 + 250

This genus lies on the dividing line between the chromatic and diatonic genera. The pyknon of 15/13 (248 cents) is virtually identical to the CI which defines the genus. The first three subgenera are the 1:1, 2:1, and 1:2 divisions respectively. Number 350 generates the 10:13:15 tritriadic scale.

D1. Characteristic Interval 15/13 248 cents

359	104/97 · 97/90 · 15/13	124 + 126 + 248	
360	78/71 · 142/135 · 15/13	163 + 86 + 248	
361	156/149 · 149/135 · 15/13	79 + 171 + 248	
362	16/15 · 15/13 · 13/12	112 + 248 + 139	SCHLESINGER
363	26/25 · 10/9 · 15/13	68 + 182 + 248	
364	256/243 · 351/320 · 15/13	90 + 160 + 248	
365	20/19 · 247/225 · 15/13	89 + 161 + 248	
366	11/10 · 15/13 · 104/99	165 + 248 + 85	
367	12/11 · 15/13 · 143/135	151 + 248 + 99	
368	46/45 · 26/23 · 15/13	38 + 212 + 248	
369	40/39 · 169/150 · 15/13	44 + 206 + 248	
370	28/27 · 39/35 · 15/13	63 + 187 + 248	
371	91/90 · 8/7 · 15/13	19 + 231 + 248	

This genus is the first indubitably diatonic genus. A pyknon, *per se*, no longer exists because the 52/45 (250 cents) is larger than one-half the perfect fourth, 4/3 (498 cents). The large composite interval in this and succeeding genera is termed the "apyknon" or non-condensation (Bryennios). Number 362 is the first tetrachord of Schlesinger's diatonic Hypodorian harmonia. Many members of this genus are reasonable approximations to Aristoxenos's soft diatonic genus, 100 + 150 + 250 cents. Others with the 15/13 medially are similar to some Byzantine tunings. Some resemble the theoretical genus 50 + 200 + 250 cents.

D2. Characteristic Interval 38/23 244 cents

372	44/41 · 41/38 · 38/33	123 + 131 + 244
373	11/10 · 20/19 · 38/33	165 + 89 + 244
374	22/21 · 21/19 · 38/33	81 + 173 + 244

This genus divides the 22/19 (254 cents).

D3. Characteristic Interval 23/20 242 cents

375	160/149 · 149/138 · 23/20	123 + 133 + 242	
376	120/109 · 218/207 · 23/20	166 + 90 + 242	
377	240/229 · 229/207 · 23/20	81 + 175 + 242	
378	8/7 · 70/69 · 23/20	231 + 25 + 242	
379	40/39 · 26/23 · 23/20	44 + 212 + 242	
380	24/23 · 23/20 · 10/9	74 + 242 + 182	SCHLESINGER

381	28/27 · 180/161 · 23/20	63 + 193 + 242	

This genus is derived from the 20:23:30 triad. The apyknon is 80/69 (256 cents), Number 380 is from Schlesinger (1932) and is described as a harmonia of "artificial formula, Phrygian". Numbers 379 and 381 make intervals of 15/13 and 7/6 respectively with their subtonics. These intervals should be contrasted with the incomposite 23/20 in the tetrachord.

D4. Characteristic Interval 31/27 239 cents

382	72/67 · 67/62 · 31/27	125 + 134 + 239	
383	108/103 · 103/93 · 31/27	82 + 177 + 239	
384	54/49 · 98/93 · 31/27	168 + 91 + 239	
385	32/31 · 9/8 · 31/27	55 + 204 + 239	

The apyknon of this genus is 36/27 (259 cents). Number 385 generates the 24:31:36 tritriadic.

D5. Characteristic Interval 39/34 238 cents

386	272/253 · 253/234 · 39/34	125 + 135 + 238	
387	408/389 · 389/351 · 39/34	83 + 178 + 238	
388	204/185 · 370/351 · 39/34	169 + 91 + 238	
389	40/39 · 39/34 · 17/15	44 + 238 + 217	

The apyknon is 136/117 (261 cents). The 39/34 interval is the 3/2's complement of 17/13 and derives from the 26:34:39 triad.

D6. Characteristic Interval 8/7 231 cents

390	14/13 · 13/12 · 8/7	128 + 139 + 231	AVICENNA
391	19/18 · 21/19 · 8/7	94 + 173 + 231	SAFIYU-D-DIN
392	21/20 · 10/9 · 8/7	84 + 182 + 231	PTOLEMY
393	28/27 · 8/7 · 9/8	63 + 231 + 204	ARCHYTAS
394	49/48 · 8/7 · 8/7	36 + 231 + 231	AL-FARABI
395	35/33 · 11/10 · 8/7	102 + 165 + 231	AVICENNA
396	77/72 · 12/11 · 8/7	116 + 151 + 231	AVICENNA
397	16/15 · 35/32 · 8/7	112 + 155 + 231	VOGEL
398	35/34 · 17/15 · 8/7	50 + 217 + 231	
399	25/24 · 8/7 · 28/25	71 + 231 + 196	
400	15/14 · 8/7 · 49/45	119 + 231 + 147	
401	40/39 · 91/80 · 8/7	44 + 223 + 231	
402	46/45 · 105/92 · 8/7	38 + 229 + 231	
403	18/17 · 119/108 · 8/7	99 + 168 + 231	
404	17/16 · 8/7 · 56/51	105 + 231 + 162	
405	34/33 · 77/68 · 8/7	52 + 215 + 231	

406 256/243 · 567/512 · 8/7 90 + 177 + 231

This genus divides the 7/6 (267 cents). The Avicenna and Al-Farabi references are from D'Erlanger. Number 390 is also given by Pachymeres (D'Erlanger 1935, 148 referring to Vincent 1847). When arranged as 13/12 · 14/13 · 8/7, it is generated by taking two successive arithmetic means. Number 394 is especially interesting as there have been reports that it was used on organs in the Middle Ages (Adler 1968; Sachs 1949), but more recent work suggests that this opinion was due to a combination of transmission errors (by copyists) and an incorrect assessment of end correction (Barbour 1950; Munxelhaus 1976). With the 49/48 medially, it is generated by the twelfth of the Greek means (Heath 1921). The scale is obviously constructed in analogy with the Pythagorean 256/243 · 9/8 · 9/8. Similar claims pro and con have been made for number 393 as well. This scale, however, appears to have been the principal tuning of the diatonic in practice from the time of Archytas (390 BCE) through that of Ptolemy (ca. 160 CE). Even Aristoxenos grudgingly mentions it (Winnington-Ingram 1932). Number 397 is from Vogel (1963) and approximates the soft diatonic. It is also found in Erickson's (1965) version of Archytas's system. Entry 399 corresponds to 3/8 + 1 1/8 + 1 tones of Aristoxenos. The Safiyu-d-Din tuning is one of his "strong" forms (2:1 division) and has 21/19 replacing the 10/9 of Ptolemy. Tetrachords 403, 404, and 405 exploit ratios of 17 and are dedicated to Larry Polansky.

D7. CHARACTERISTIC INTERVAL 256/225 223 CENTS

407	150/139 · 139/128 · 256/225	132 + 143 + 223
408	225/214 · 107/96 · 256/225	87 + 188 + 223
409	225/203 · 203/192 · 256/225	78 + 96 + 223
410	25/24 · 9/8 · 256/225	71 + 204 + 223

The apyknon is the augmented second, 75/64 (275 cents). Number 410 is the generator of the 64:75:96 tritriadic and a good approximation to Aristoxenos's 3/8 + 1 1/8 + 1 tone when reordered so that the 9/8 is uppermost.

D8. CHARACTERISTIC INTERVAL 25/22 221 CENTS

411	176/163 · 163/150 · 25/22	133 + 144 + 221
412	132/119 · 238/225 · 25/22	179 + 97 + 221
413	264/251 · 251/225 · 25/22	87 + 189 + 221
414	16/15 · 11/10 · 25/22	112 + 165 + 221
415	88/81 · 27/25 · 25/22	143 + 133 + 221

416	22/21 · 25/22 · 28/25	81 + 221 + 196	
417	28/27 · 198/175 · 25/22	63 + 214 + 221	
418	26/25 · 44/39 · 25/22	68 + 209 + 221	

This is an experimental genus whose apyknon is 88/75 (277 cents). Number 416 is a fair approximation of Aristoxenos's 3/8 + 1 1/8 + 1 tones, and number 411 is close to a hypothetical 11/16+11/16+1 1/8 tones.

D9. Characteristic Interval 92/81 220 cents

419	27/25 · 25/23 · 92/81	133 + 144 + 220	
420	81/77 · 77/69 · 92/81	88 + 190 + 220	
421	81/73 · 73/69 · 92/81	180 + 98 + 220	
422	24/23 · 9/8 · 92/81	74 + 204 + 220	
423	27/26 · 26/23 · 92/81	66 + 212 + 220	

This genus divides the 27/23 (278 cents) and is derived from the 18:23:27 triad. Number 422 is the tritriadic generator, and is an approximation to Aristoxenos's 3/8+1 1/8+1 tones (4.5+13.5+12 "parts") when reordered.

D10. Characteristic Interval 76/67 218 cents

424	67/62 · 62/57 · 76/67	134 + 146 + 218	
425	201/181 · 181/171 · 76/67	181 + 98 + 218	
426	201/191 · 191/171 · 76/67	88 + 191 + 218	
427	256/243 · 76/67 · 5427/4864	90 + 218 + 190	EULER

This complex genus is expanded from number 427, which is called "old chromatic" in Euler's text (Euler [1739] 1960, 177). The tuning is clearly diatonic, however, and must be in error. It may have been intended to represent Boethius's 19/16 (76/64) chromatic. The apyknon is 67/57 (280 cents).

D11. Characteristic Interval 17/15 217 cents

428	40/37 · 37/34 · 17/15	135 + 146 + 217	
429	10/9 · 18/17 · 17/15	182 + 99 + 217	KORNERUP
430	20/19 · 19/17 · 17/15	89 + 192 + 217	PTOLEMY
431	15/14 · 56/51 · 17/15	119 + 162 + 217	
432	80/77 · 77/68 · 17/15	66 + 215 + 217	
433	12/11 · 55/51 · 17/15	151 + 131 + 217	
434	120/109 · 109/102 · 17/15	166 + 115 + 217	
435	120/113 · 113/102 · 17/15	104 + 177 + 217	
436	24/23 · 115/102 · 17/15	74 + 208 + 217	
437	160/153 · 9/8 · 17/15	77 + 204 + 217	

This genus divides the 20/17 (281 cents). Number 429 is Kornerup's (1934,

10) Lydian. Genus number 430 is Ptolemy's interpretation of Aristoxenos's intense diatonic, 6 + 12 + 12 "parts" (Wallis 1682, 172). Kornerup refers to it as Dorian. Number 432 is a hypothetical Ptolemaic interpretation of 4.5 + 13.5 + 12 "parts", a mixed chromatic and diatonic genus not in Ptolemy. Number 437 generates the 34:40:51 triad and tritriadic. The remaining divisions are experimental neo-Aristoxenian genera with a constant upper interval of 12 "parts."

D12. Characteristic Interval 112/99 214 cents

438	66/61 · 61/56 · 112/99	136 + 148 + 214	
439	99/94 · 47/42 · 112/99	90 + 195 + 214	
440	99/89 · 89/84 · 112/99	184 + 100 + 214	
441	10/9 · 297/280 · 112/99	182 + 102 + 214	
442	22/21 · 9/8 · 112/99	81 + 204 + 214	

This very complex genus divides the 33/28 (284 cents). Number 442 generates the 22:28:33 tritriadic and its conjugate.

D13. Characteristic Interval 44/39 209 cents

443	12/11 · 13/12 · 44/39	151 + 139 + 209	YOUNG
444	39/35 · 35/33 · 44/39	187 + 102 + 209	
445	39/37 · 37/33 · 44/39	91 + 198 + 209	
446	44/39 · 9/8 · 104/99	209 + 204 + 85	

The first division is William Lyman Young's "exquisite 3/4-tone Hellenic lyre" (Young 1961, 5). The apyknon is 13/11 (289 cents). Number 446 generates the 22:26:33 tritriadic scale.

D14. Characteristic Interval 152/135 205 cents

447	90/83 · 83/76 · 152/135	140 + 153 + 205	
448	135/128 · 64/57 · 152/135	92 + 201 + 205	
449	135/121 · 121/114 · 152/135	190 + 103 + 205	
450	20/19 · 9/8 · 152/135	89 + 204 + 205	

This genus derives from the 30:38:45 triad and divides its upper interval, 45/38 (293 cents). Number 450 generates the 30:38:45 tritriadic and its conjugate.

D15. Characteristic Interval 9/8 204 cents

451	64/59 · 59/54 · 9/8	141 + 153 + 204	SAFIYU-D-DIN
452	48/43 · 86/81 · 9/8	190 + 104 + 204	SAFIYU-D-DIN
453	96/91 · 91/81 · 9/8	93 + 202 + 204	
454	256/243 · 9/8 · 9/8	90 + 204 + 204	PYTHAGORAS?

455	16/15 · 9/8 · 10/9	112 + 204 + 182	PTOLEMY, DIDYMOS
456	2187/2048 · 65536/59049 · 9/8	114 + 180 + 204	ANONYMOUS
457	9/8 · 12/11 · 88/81	204 + 151 + 143	AVICENNA
458	13/12 · 9/8 · 128/117	139 + 204 + 156	AVICENNA
459	14/13 · 9/8 · 208/189	128 + 204 + 166	AVICENNA
460	9/8 · 11/10 · 320/297	204 + 165 + 129	AL-FARABI
461	9/8 · 15/14 · 448/405	204 + 119 + 175	
462	9/8 · 17/16 · 512/459	204 + 105 + 189	
463	9/8 · 18/17 · 272/243	204 + 99 + 195	
464	9/8 · 19/18 · 64/57	204 + 94 + 201	
465	56/51 · 9/8 · 68/63	162 + 204 + 132	
466	9/8 · 200/189 · 28/25	204 + 98 + 196	
467	184/171 · 9/8 · 76/69	127 + 204 + 167	
468	32/29 · 9/8 · 29/27	170 + 204 + 124	
469	121/108 · 9/8 · 128/121	197 + 204 + 97	PARTCH
470	9/8 · 4096/3645 · 135/128	204 + 202 + 92	
471	9/8 · 7168/6561 · 243/224	204 + 153 + 141	
472	35/32 · 1024/945 · 9/8	204 + 139 + 204	

The apyknon of this genus is 32/27 (294 cents). Numbers 451 and 452 are Safiyu-d-Din's weak and strong forms of the division, respectively. The attribution of the tetrachord number 454 to Pythagoras is questionable, though traditional—the diatonic scale in "Pythagorean" intonation antedates him by a millennium or so in the Near East (Duchesne-Guillemin 1963, 1969). The earliest reference to this scale in a European language is in Plato's *Timaeus*. Number 455 is attributed to both Ptolemy and Didymos because their historically important definitions differed in the order of the intervals. Ptolemy's is the order shown; Didymos placed the 9/8 at the top. Ptolemy's order generates the major mode in just intonation. Its retrograde, 10/9 · 9/8 · 16/15, yields the natural minor and new scale of Redfield (1928). Number 456 is a "Pythagorean" form extracted from the anonymous treatise in D'Erlanger (1939). In reverse order, it appears in the Turkish scales of Palmer (1967?). Numbers 457–460 are also from D'Erlanger. Numbers 457 and 458 generate the 18:22:27 and 26:32:39 tritriadics and their conjugates. These and the tetrachord from Al-Farabi, number 459, resemble modern Islamic tunings (Sachs 1943, 283). Numbers 464 and 465 generate the 16:19:24 and the 14:17:21 tritriadics. In theory, any tetrachord containing a 9/8 generates a tritriadic and its conjugate, but in practice the majority are not very consonant. Examples are numbers 467 and 468 which generate the 38:46:57 and 24:29:36 tritriadics with mediants of 23/19 and 29/24. Number 469 is an adventitious tetrachord from Partch (1974, 165). Numbers 470–472 are from chapter 4. The last two resemble some of the Islamic tunings of the Middle Ages. The remaining tunings are proposed approximations to Islamic or syntonic diatonic tetrachords.

D16. CHARACTERISTIC INTERVAL 160/143 194 CENTS

473	11/10 · 13/12 · 160/143	165 + 139 + 194	AL-FARABI

This tetrachord is from Al-Farabi (D'Erlanger 1930, 112). It did not seem worthwhile to explore this genus further because the ratios would be complex and often larger than 160/143 itself.

D17. CHARACTERISTIC INTERVAL 10/9 182 CENTS

474	12/11 · 11/10 · 10/9	151 + 165 + 182	PTOLEMY
475	10/9 · 10/9 · 27/25	182 + 182 + 133	AL-FARABI
476	10/9 · 13/12 · 72/65	182 + 139 + 177	AVICENNA

The apyknon is 6/5 and the majority of potential divisions have intervals larger than the 10/9. Number 474 is Ptolemy's homalon or equable diatonic, a scale which has puzzled theorists, but which seems closely related to extant tunings in the Near East. Ptolemy described it as sounding rather foreign and rustic. Could he have heard it or something similar and written it down in the simplest ratios available? It certainly sounds fine, perhaps a bit like 7-tone equal temperament with perfect fourths and fifths. The Avicenna and Al-Farabi references are from D'Erlanger (1935), and Ptolemy (Wallis 1682).

Reduplicated tetrachords

These genera are arranged by the reduplicated interval in descending order of size.

477	11/10 · 11/10 · 400/363	165 + 165 + 168		R1
478	12/11 · 12/11 · 121/108	151 + 151 + 197	AVICENNA	R2
479	13/12 · 13/12 · 192/169	139 + 139 + 221	AVICENNA	R3
480	14/13 · 14/13 · 169/147	128 + 128 + 241	AVICENNA	R4
481	15/14 · 15/14 · 784/675	119 + 119 + 259	AVICENNA	R5
482	2187/2048 · 16777216/14348907 · 2187/2048	114 + 271 + 114	PALMER	R6
483	17/16 · 17/16 · 1024/867	105 + 105 + 288		R7
484	18/17 · 18/17 · 289/243	99 + 99 + 300		R8
485	256/243 · 256/243 · 19688/16384	90 + 90 + 318		R9
486	22/21 · 147/121 · 22/21	81 + 337 + 81		R10

487	25/24 · 25/24 · 768/625	71 + 71 + 357	RI1
488	28/27 · 28/27 · 243/196	63 + 63 + 372	RI2
489	34/33 · 34/33 · 363/289	52 + 52 + 395	RI3
490	36/35 · 36/25 · 1225/972	49 + 49 + 401	RI4
491	40/39 · 40/39 · 507/400	44 + 44 + 410	RI5
492	46/45 · 46/45 · 675/529	38 + 38 + 422	RI6

While a number of other small intervals could be used to construct analogous genera, the ones given here seem the most important and most interesting. Number 477 is an approximation in just intonation to the equally tempered division of the 4/3. See number 722 for the semi-tempered version. The Avicenna genera are from vol. 2, pages 122–123 and page 252 of D'Erlanger. The Palmer genus is from his booklet on Turkish music (1967?). This genus is very close to Helmholtz's chromatic 16/15 · 75/64 · 16/15. The 18/17 genus is also nearly equally tempered and is inspired by Vincenzo Galilei's lute fretting (Barbour 1951, 57). Number 486 is nearly equal to 1/1 $\pi/3$ $4/\pi$ 4/3, a theoretical genus using intervals of 11 to approximate intervals of π. Numbers 487 and 488 come from Winnington-Ingram's (1932) suggestion that Aristoxenos's soft and hemiolic chromatics were somewhat factitious genera resulting from the duplication of small, but known, intervals. The remaining tetrachords are in the spirit of Avicenna and Al-Farabi.

Miscellaneous tetrachords

The tetrachords in this section are those that were discovered in the course of various theoretical studies but which were not judged to be of sufficient interest to enter in the Main Catalog. Many of these genera have unusual CIs which were not thought worthy of further study. The fourth and fifth columns give the ratio of the pyknon or apyknon and its value in cents.

493	176/175 · 175/174 · 29/22	10 + 10 + 478	88/87	20	M1
494	25/19 · 931/925 · 148/147	475 + 11 + 12	76/75	23	M2

This tetrachord is generated by the second of the summation procedures of chapter 5.

495	128/127 · 127/126 · 21/16	14 + 14 + 471	64/63	27	M3
496	21/16 · 656/651 · 124/123	471 + 13 + 14	64/63	27	M4

Another summation tetrachord from chapter 4.

497	104/103 · 103/102 · 17/13	17 + 17 + 464	52/51	34	M5
498	17/13 · 429/425 · 100/99	464 + 16 + 17	52/51	34	M6

Another summation tetrachord from chapter 4.

499	98/97 · 97/96 · 64/49	18 + 18 + 462	49/48	36	M7
500	92/91 · 91/90 · 30/23	19 + 19 + 460	46/45	38	M8
501	90/89 · 89/88 · 176/135	19 + 20 + 459	45/44	39	M9
502	88/87 · 87/86 · 43/33	20 + 20 + 458	44/43	40	M10
503	86/85 · 85/84 · 56/43	20 + 20 + 457	43/42	41	M11
504	84/83 · 83/82 · 82/63	21 + 21 + 456	42/41	42	M12
505	82/81 · 81/80 · 160/123	21 + 22 + 455	41/40	43	M13

These genera contain intervals which are probably too small for use in most music. However, Harry Partch and Julián Carrillo, among others, have used intervals in this range.

506	13/10 · 250/247 · 76/74	454 + 21 + 23	40/39	44	M14

Another summation tetrachord from chapter 4.

507	78/77 · 77/76 · 152/117	22 + 23 + 453	39/38	45	M15
508	76/75 · 76/75 · 74/57	23 + 23 + 452	38/37	46	M16
509	74/73 · 73/72 · 48/31	24 + 24 + 451	37/36	47	M17
510	70/69 · 69/68 · 136/105	25 + 25 + 448	35/34	50	M18
511	22/17 · 357/352 · 64/63	446 + 24 + 27	34/33	52	M19

Another summation tetrachord from chapter 4.

512	58/57 · 57/56 · 112/87	30 + 31 + 437	29/28	61	M20
513	87/80 · 43/42 · 112/87	20 + 41 + 437	29/28	61	M21
514	87/85 · 85/84 · 112/87	40 + 20 + 437	29/28	61	M22

The preceding are a set of hyperenharmonic genera which divide the dieses between 40/39 and 28/27. Similar but simpler genera will be found in the Main Catalog. Small intervals in this range are clearly perceptible, but have been rejected by most theoreticians, ancient and modern.

515	68/53 · 53/52 · 52/51	431 + 33 + 34	53/51	67	M23
516	136/133 · 133/130 · 65/51	34 + 34 + 420	68/65	78	M24
517	68/67 · 67/65 · 65/51	26 + 52 + 420	68/65	78	M25
518	34/33 · 66/65 · 65/51	52 + 26 + 420	68/65	78	M26
519	68/67 · 67/54 · 18/17	26 + 373 + 99	72/76	125	M27
520	25/24 · 32/31 · 31/25	71 + 55 + 372	100/93	126	M28
521	68/55 · 55/54 · 18/17	367 + 32 + 99	55/51	131	M29
522	68/67 · 67/63 · 21/17	26 + 107 + 366	68/63	132	M30
523	68/65 · 65/63 · 21/17	78 + 54 + 366	68/63	132	M31
524	36/35 · 256/243 · 315/256	49 + 90 + 359	1024/945	139	M32
525	64/63 · 16/15 · 315/256	27 + 112 + 359	1024/945	139	M33

Numbers 524 and 525 are from Vogel's PIS tuning of chapter 6.

526	64/63 · 2187/2048 · 896/729	27 + 114 + 357	243/224	141	M34
527	36/35 · 135/128 · 896/729	49 + 92 + 357	243/224	141	M35

This tuning is a close approximation to one produced by the eighth mean (Heath 1921) of chapter 4. It also occurs in Erickson's analysis of Archytas's system and in Vogel's tuning (chapter 6 and Vogel 1963, 197).

528	28/27·2187/1792·256/243	63 + 345 + 90	7168/6561	153	M36

This tetrachord appears in Erickson's commentary on Archytas's system with trite synemmenon (112/81, B♭-) added.

529	16/15·2240/2187·2187/1792	112 + 41 + 345	7168/6561	153	M37
530	28/27·128/105·135/128	63 + 343 + 92	35/32	141	M38

Numbers 528–530 are from Vogel's PIS tuning of chapter 6.

531	17/16·32/31·62/51	105 + 55 + 338	34/31	160	M39
532	20/19·57/47·47/45	89 + 334 + 75	188/171	164	M40

Number 532 is a possible Byzantine chromatic.

533	360/349·349/327·109/90	54 + 113 + 332	120/109	166	M41
534	24/23·115/109·109/90	74 + 94 + 332	120/109	166	M42

Number 534 is a hypothetical Ptolemaic interpretation of 5 + 6 + 19 "parts", after Macran (1902).

535	240/229 · 229/218 · 109/90	81 + 85 + 332	120/109	166	M43
536	19/18 · 24/23 · 23/19	94 + 74 + 330	76/69	167	M44
537	15/14 · 36/35 · 98/81	119 + 49 + 330	54/49	168	M45

Number 537 occurs in Other Music's gamelan tuning (Henry S. Rosenthal, personal communication).

538	28/27 · 16/15 · 135/112	63 + 112 + 323	448/405	175	M46
539	24/23 · 115/96 · 16/15	74 + 313 + 112	128/115	185	M47

A Ptolemaic interpretation of Xenakis's 5 + 19 + 6 "parts" (1971).

540	256/243 · 243/230 · 115/96	90 + 95 + 313	128/115	185	M48
541	68/67 · 67/56 · 56/51	26 + 310 + 162	224/201	88	M49
542	68/57 · 19/18 · 18/17	305 + 94 + 99	19/17	193	M50
543	15/14 · 266/255 · 68/57	119 + 73 + 305	19/17	193	M51
544	256/243 · 243/229 · 229/192	90 + 103 + 305	256/192	193	M52
545	32/31 · 13/12 · 31/26	55 + 139 + 304	104/93	194	M53
546	240/227 · 227/214 · 107/90	96 + 102 + 300	120/107	199	M54
547	360/347 · 347/321 · 107/90	64 + 135 + 300	120/107	199	M55

This genus is related to (Ps.)-Philolaus's division as 6.5 + 6.5 + 17 "parts". See also chapter 4.

548	7168/6561 · 36/35 · 1215/1024	153 + 49 + 296	4096/3645	202	M56

549 16/15 · 1215/1024 · 256/243 112 + 296 + 90 4096/3635 202 M57

550 28/27 · 1024/945 · 1215/1024 63 + 139 + 296 4096/3635 202 M58

Numbers 548–550 are from Vogel's PIS tuning of chapter 6.

551 120/113 · 113/106 · 53/45 104 + 111 + 283 60/53 215 M59

552 180/173 · 173/159 · 53/45 69 + 146 + 283 60/53 215 M60

553 90/83 · 166/159 · 53/45 140 + 75 + 283 60/53 215 M61

554 24/23 · 115/106 · 53/45 74 + 141 + 283 60/53 215 M62

Number 554 is a hypothetical Ptolemaic interpretation of 5 + 9 + 16 "parts." The others, numbers 551, 552, and 553 are 1:1, 1:2 and 2:1 divisions of the pyknon.

555 34/29 · 58/57 · 19/17 275 + 30 + 193 58/51 223 M63

556 10/9 · 117/100 · 40/39 182 + 272 + 44 400/351 226 M64

557 120/113 · 113/97 · 97/90 104 + 264 + 130 388/339 234 M65

This genus is a Ptolemaic interpretation of Xenakis's 7 + 16 + 7 "parts."

558 13/12 · 55/52 · 64/55 139 + 97 + 262 55/48 236 M66

This genus is generated by the second ratio mean of chapter 4.

559 68/65 · 65/56 · 56/51 78 + 258 + 162 224/195 240 M67

560 12/11 · 297/256 · 256/243 151 + 257 + 90 1024/891 241 M68

561 28/27 · 81/70 · 10/9 63 + 253 + 182 280/243 245 M69

This tetrachord is also found in Erickson's article on Archytas's system with trite synemmenon (112/81, B♭-) added. It also occurs in Vogel's PIS tuning of chapter 6.

562 81/70 · 2240/2187 · 9/8 253 + 41 + 204 280/243 245 M70

563 81/70 · 256/243 · 35/32 253 + 90 + 155 280/243 245 M71

564 135/128 · 7168/6561 · 81/70 92 + 153 + 253 280/243 245 M72

These three tetrachords are from Vogel's PIS tuning of chapter 6.

565 60/59 · 59/51 · 17/15 29 + 252 + 217 68/59 246 M73

566 40/37 · 37/32 · 16/15 135 + 251 + 112 128/111 247 M74

This is a Ptolemaic interpretation of Athanasopoulos's 9 + 15 + 6 "parts."

567 16/15 · 280/243 · 243/224 112 + 245 + 141 81/70 253 M75

568 36/35 · 9/8 · 280/243 49 + 204 + 245 81/70 253 M76

569 8/7 · 81/80 · 280/243 231 + 22 + 245 81/70 253 M77

These three tetrachords are from Vogel's PIS tuning of chapter 6.

570 46/45 · 132/115 · 25/22 38 + 239 + 221 115/99 259 M78

571 16/15 · 12/11 · 55/48 112 + 151 + 236 64/55 262 M79

This is an approximation to the soft diatonic of Aristoxenos, 1/2 + 3/4 + 1 1/4 tones, 6 + 9 + 15 "parts."

572	10/9 · 63/55 · 22/21	182 + 235 + 81	220/189	263	M80

This is another tetrachord from Partch ([1949] 1974, 165), presented as an approximation to a tetrachord of the "Ptolemaic sequence," or major mode in 5-limit just intonation.

573	30/29 · 116/103 · 103/90	59 + 206 + 234	120/103	264	M81
574	360/343 · 343/309 · 103/90	84 + 181 + 234	120/103	264	M82
575	40/39 · 143/125 · 25/22	44 + 233 + 221	500/429	265	M83
576	68/65 · 65/57 · 19/17	78 + 227 + 193	76/65	271	M84
577	256/243 · 729/640 · 10/9	90 + 225 + 182	2560/2187	273	M85
578	30/29 · 58/51 · 17/15	59 + 223 + 217	34/29	275	M86
579	23/21 · 14/13 · 26/23	158 + 128 + 212	46/39	286	M87
580	23/22 · 44/39 · 26/23	77 + 209 + 212	46/39	286	M88
581	14/13 · 260/231 · 11/10	128 + 205 + 165	77/65	293	M89
582	4096/3645 · 35/32 · 243/224	202 + 155 + 141	1215/1024	296	M90

From Vogel's PIS tuning of chapter 6.

583	38/35 · 35/32 · 64/57	142 + 155 + 201	19/16	298	M91
584	19/17 · 17/16 · 64/57	193 + 105 + 201	19/16	298	M92
585	11/10 · 95/88 · 64/57	165 + 135 + 201	19/16	298	M93

The apyknon of genera numbers 583–585 is 19/16. The 1:2 division is listed as D15 (9/8), number 464.

586	240/221 · 221/202 · 101/90	143 + 156 + 200	120/101	298	M94
587	15/14 · 112/101 · 101/90	119 + 179 + 200	120/101	298	M95
588	120/113 · 113/101 · 101/90	104 + 194 + 200	120/101	298	M96
589	533/483 · 575/533 · 28/25	171 + 131 + 196	25/21	302	M97

A mean tetrachord of the first kind from chapter 4.

590	19/17 · 85/76 · 16/15	193 + 194 + 112	304/255	304	M98
591	19/17 · 1156/1083 · 19/17	193 + 113 + 193	68/57	305	M99

Two tetrachords from Thomas Smith (personal communication, 1989).

592	68/63 · 21/19 · 19/17	132 + 173 + 193	68/57	305	M100
593	10/9 · 108/97 · 97/90	182 + 186 + 130	97/90	368	M101

Tetrachords in equal temperament

The tetrachords listed in this section of the Catalog are the genera of Aristoxenos and other writers in this tradition (chapter 3). Included also are those genera which appear as vertices in the computations of Rothenberg's propriety function and other descriptors, and various neo-Aristoxenian genera. These are all divisions of the tempered fourth (500 cents).

The "parts" of the fourth used to describe the scales of Aristoxenos are, in fact, the invention of Cleonides, a later Greek writer, as Aristoxenos spoke only of fractional tones. The invention has proved both useful and durable, for not only the later classical writers, but also the Islamic theorists and the modern Greek Orthodox church employ the system, though the former have often doubled the number to avoid fractional parts in the hemiolic chromatic and a few other genera.

Until recently, the Greek church has used a system of 28 parts to the fourth (Tiby 1938), yielding a theoretical octave of 68 (28 + 12 + 28) tones rather than the 72 (30 + 12 + 30 = 72) or 144 (60 + 24 + 60 = 144 in the hemiolic chromatic and rejected genera) of the Aristoxenians. The 68-tone equal temperament has a fourth of only 494 cents.

Note that a number of the Orthodox liturgical tetrachords are meant to be permuted in the formation of the different modes (echoi). This operation may be applied to the historical and neo-Aristoxenian ones as well.

ARISTOXENIAN STYLE TETRACHORDS

594	2 + 2 + 26	33 + 33 + 433	CHAPTER 4	T1
595	2.5 + 2.5 + 25	42 + 42 + 417	CHAPTER 4	T2
596	2 + 3 + 25	33 + 50 + 417	CHAPTER 4	T3
597	3 + 3 + 24	50 + 50 + 400	ARISTOXENOS	T4
598	2 + 4 + 24	33 + 67 + 400	CHAPTER 4	T5
599	2 + 5 + 23	33 + 83 + 383	CHAPTER 4	T6
600	7/3 + 14/3 + 23	39 + 78 + 383	CHAPTER 4	T7
601	4 + 3 + 23	67 + 50 + 383	CHAPTER 3	T8
602	3.5 + 3.5 + 23	58 + 58 + 383	CHAPTER 4	T9
603	2 + 6 + 22	33 + 100 + 367	CHAPTER 4	T10
604	4 + 4 + 22	66 + 66 + 367	ARISTOXENOS	T11
605	8/3 + 16/3 + 22	44 + 89 + 367	CHAPTER 4	T12
606	3 + 5 + 22	50 + 83 + 367	CHAPTER 4	T13
607	4.5 + 3.5 + 22	75 + 58 + 367	ARISTOXENOS	T14
608	2 + 7 + 21	33 + 117 + 350	CHAPTER 4	T15
609	3 + 6 + 21	50 + 100 + 350	CHAPTER 4	T16
610	4.5 + 4.5 + 21	75 + 75 + 350	ARISTOXENOS	T17
611	4 + 5 + 21	67 + 83 + 350	CHAPTER 4	T18
612	6 + 3 + 21	100 + 50 + 350	ARISTOXENOS	T19
613	6 + 20 + 4	100 + 333 + 67	SAVAS	T20
614	10/3 + 20/3 + 20	56 + 111 + 333	CHAPTER 4	T21

615	5 + 5 + 20	83 + 83 + 334	CHAPTER 4	T22
616	5.5 + 5.5 + 19	92 + 92 + 317	CHAPTER 4	T23
617	11/3 + 22/3 + 19	61 + 122 + 317	CHAPTER 4	T24
618	5 + 19 + 6	83 + 317 + 100	XENAKIS	T25
619	5 + 6 + 19	83 + 100 + 317	MACRAN	T26
620	2 + 10 + 18	33 + 167 + 300	CHAPTER 4	T27
621	3 + 9 + 18	50 + 150 + 300	CHAPTER 4	T28
622	4 + 8 + 18	67 + 133 + 300	ARISTOXENOS	T29
623	4.5 + 7.5 + 18	75 + 125 + 300	CHAPTER 4	T30
624	6 + 6 + 18	100 + 100 + 300	ARISTOXENOS	T31
625	5 + 7 + 18	83 + 117 + 300	CHAPTER 4	T32
626	6 + 18 + 6	100 + 300 + 100	ATHANASOPOULOS	T33
627	13/3 + 26/3 + 17	72 + 144 + 283	CHAPTER 4	T34
628	6.5 + 6.5 + 17	108 + 108 + 283	CHAPTER 4	T35
629	2 + 16 + 12	33 + 267 + 200	CHAPTER 4	T36
630	14/3 + 28/3 + 16	78 + 156 + 267	CHAPTER 4	T37
631	5 + 9 + 16	83 + 150 + 267	WINNINGTON-INGRAM	T38
632	8 + 16 + 6	133 + 267 + 100	SAVAS	T39
633	7 + 16 + 7	117 + 267 + 117	XENAKIS; CHAP. 4	T40
634	2 + 13 + 15	33 + 217 + 250	CHAPTER 4	T41
635	3 + 12 + 15	50 + 200 + 250	CHAPTER 4	T42
636	4 + 11 + 15	67 + 183 + 250	CHAPTER 4	T43
637	5 + 10 + 15	83 + 167 + 250	CHAPTER 4	T44
638	6 + 9 + 15	100 + 150 + 250	ARISTOXENOS	T45
639	7 + 8 + 15	117 + 133 + 250	CHAPTER 4	T46
640	7.5 + 7.5 + 15	125 + 125 + 250	CHAPTER 4	T47
641	9 + 15 + 6	150 + 250 + 100	ATHANASOPOULOS	T48
642	2 + 14 + 14	33 + 233 + 233	CHAPTER 4	T49
643	4 + 14 + 12	67 + 233 + 200	ARISTOXENOS	T50
644	5 + 11 + 14	83 + 183 + 233	WINNINGTON-INGRAM	T51
645	16/3 + 32/3 + 14	89 + 178 + 233	CHAPTER 4	T52
646	8 + 8 + 14	133 + 133 + 233	CHAPTER 4	T53
647	4.5 + 13.5 + 12	75 + 225 + 200	ARISTOXENOS	T54
648	5 + 12 + 13	83 + 200 + 217	CHAPTER 4	T55
649	4 + 13 + 13	67 + 217 + 217	CHAPTER 4	T56
650	17/3 + 34/3 + 13	94 + 189 + 217	CHAPTER 4	T57
651	8.5 + 8.5 + 13	142 + 142 + 217	CHAPTER 4	T58

652	6 + 12 + 12	100 + 200 + 200	ARISTOXENOS	T59

Savas, Xenakis and Athanasopoulos all give permutations of this tetrachord in their lists of Orthodox church forms.

653	12 + 11 + 7	200 + 183 + 117	XENAKIS	T60

Xenakis (1971) permits several permutations of this approximation to Ptolemy's intense diatonic.

654	10 + 8 + 12	167 + 133 + 200	SAVAS	T61

The form 8 + 12 + 10 is Savas's "Barys diatonic" (Savas 1965).

655	12 + 9 + 9	200 + 150 + 150	AL-FARABI; CH. 4	T62
656	8 + 11 + 11	133 + 183 + 183	CHAPTER 4	T63

This tuning is close to 27/25 · 10/9 · 10/9.

657	9.5 + 9.5 + 11	158 + 158 + 183	CHAPTER 4	T64
658	10 + 10 + 10	166 + 167 + 167	AL-FARABI	T65

Tiby's Greek Orthodox tetrachords of 28 parts to the fourth of 494 cents.

659	12 + 13 + 3	212 + 229 + 53	TIBY	T66
660	12 + 5 + 11	212 + 88 + 194	TIBY	T67
661	12 + 9 + 7	212 + 159 + 124	TIBY	T68
662	9 + 12 + 7	159 + 212 + 124	TIBY	T69

See Tiby (1938) for numbers 659–662.

TEMPERED TETRACHORDS IN CENTS

663	22.7 + 22.7 + 454.5	CHAPTER 5	T70
664	37.5 + 37.5 + 425	CHAPTER 5	T71
665	62.5 + 62.5 + 375	CHAPTER 5	T72

Tetrachord numbers 663– 665 are categorical limits in the classification scheme of 5-9.

666	95 + 115 + 290		T73

This tetrachord was designed to fill a small gap in tetrachordal space. See 9-4, 9-5, and 9-6.

667	89 + 289 + 122	CHAPTER 5	T74
668	87.5 + 287.5 + 125	CHAPTER 5	T75
669	83.3 + 283.3 + 133.3	CHAPTER 5	T76
670	75 + 275 + 150	CHAPTER 5	T77
671	100 + 275 + 125	CHAPTER 5	T78
672	55 + 170 + 275		T79

This tetrachord was designed to fill a small gap in tetrachordal space.

673	66.7 + 266.7 + 166.7	CHAPTER 5	T80
674	233.3 + 16.7 + 250	CHAPTER 5	T81

675 225 + 25 + 250 CHAPTER 5 T82

676 66.7 + 183.3 + 250 CHAPTER 5 T83

677 75 + 175 + 250 CHAPTER 5 T84

678 125 + 125 + 250 CHAPTER 5 T85

679 105 + 145 + 250 T86

680 110 + 140 + 250 T87

Tetrachord numbers 679 and 680 fill possible gaps in tetrachordal space.

681 87.5 + 237.5 + 175 CHAPTER 5 T88

682 233.3 + 166.7 + 100 CHAPTER 5 T89

683 212.5 + 62.5 + 225 CHAPTER 5 T90

684 225 + 75 + 200 CHAPTER 5 T91

685 225 + 175 + 100 CHAPTER 5 T92

686 87.5 + 187.5 + 225 CHAPTER 5 T93

687 212.5 + 162.5 + 125 CHAPTER 5 T94

688 100 + 187.5 + 212.5 CHAPTER 5 T95

689 212.5 + 137.5 + 150 CHAPTER 5 T96

690 200 + 125 + 175 CHAPTER 5 T97

691 145 + 165 + 190 T98

This tetrachord was designed to fill a small gap in tetrachordal space.

Semi-tempered tetrachords

The tetrachords in this section contain both just and tempered intervals. Two of these genera are literal interpretations of late Classical tuning theory. A number are based on the assumption that Aristoxenos intended to divide the perfect fourth (4/3), a rather doubtful hypothesis. The remainder are mean tetrachords from chapter 4 with medial 9/8. Formally, these latter tetrachords are generators of tritriadic scales. In all cases they span a pure 4/3.

692 $16/(9\sqrt{3}) \cdot 16/(9\sqrt{3}) \cdot 81/64$ 45 + 45 + 408 S1

Number 692 is Barbera's (1978) literal interpretation of Nicomachos's enharmonic as 1/2 semitone + 1/2 semitone + ditone, where the 1/2 semitone is the square root of 256/243, also written as $16 \cdot \sqrt{3} / 27$.

693 $1.26376 \cdot 1.05321 \cdot 1.00260$ 405 + 88 + 4 S2

This mean tetrachord of the second kind is generated by mean 9.

694 $(4/3)^{1/10} \cdot (4/3)^{1/10} \cdot (4/3)^{8/10}$ 50 + 50 + 398 S3

This tetrachord is a literal interpretation of Aristoxenos's enharmonic under Barbera's (1978) assumption that Aristoxenos's meant the perfect fourth 4/3. In Cleonides's cipher, it is 3 + 3 + 24 parts.

695 $(4/3)^{2/15} \cdot (4/3)^{2/15} \cdot (4/3)^{11/15}$ 66 + 66 + 365 s4

This tetrachord is a semi-tempered interpretation of Aristoxenos's soft chromatic. In Cleonides's cipher, it is 4 + 4 + 22 parts.

696 $(4/3)^{3/20} \cdot (4/3)^{7/60} \cdot (4/3)^{11/15}$ 75 + 58 + 365 s5

This tetrachord is a semi-tempered interpretation of a genus rejected by Aristoxenos. It somewhat resembles Archytas's enharmonic. In Cleonides's cipher, it is 4.5 + 3.5 + 22 parts.

697 $(4/3)^{3/20} \cdot (4/3)^{3/20} \cdot (4/3)^{7/10}$ 75 + 75 + 349 s6

This tetrachord is a semi-tempered interpretation of Aristoxenos's hemiolic chromatic. In Cleonides's cipher, it is 4.5 + 4.5 + 21 parts.

698 $(4/3)^{1/5} \cdot (4/3)^{1/10} \cdot (4/3)^{7/10}$ 100 + 50 + 349 s7

This tetrachord is a semi-tempered interpretation of a genus rejected by Aristoxenos. In Cleonides's cipher, it is 6 + 3 + 21 parts.

699 1.21677 · 1.03862 · 1.05505 340 + 66 + 93 s8

This mean tetrachord of the first kind is generated by mean 9.

700 $(4/3)^{1/5} \cdot (4/3)^{1/5} \cdot (4/3)^{3/5}$ 100 + 100 + 299 s9

This tetrachord is a semi-tempered interpretation of Aristoxenos's intense chromatic. In Cleonides's cipher, it is 6 + 6 + 18 parts.

701 $(4/3)^{2/15} \cdot (4/3)^{4/15} \cdot (4/3)^{3/5}$ 66 + 133 + 299 s10

This tetrachord is a semi-tempered interpretation of a genus rejected by Aristoxenos. It closely resembles Archytas's chromatic In Cleonides's cipher, it is 4 + 8 + 18 parts.

702 $3\sqrt{2}/4 \cdot 3\sqrt{2}/4 \cdot 32/27$ 102 + 102 + 294 s11

This tetrachord is implied by writers such as Thrasyllus who did not give numbers for the chromatic, but stated only that it contained a 32/27 and a 1:1 pyknon (Barbera 1978). The semitones are the square root of 9/8.

703 1.18046 · 1.06685 · 1.05873 287 + 112 + 99 s12

This mean tetrachord of the second kind is generated by mean 5.

704 1.05956 · 1.06763 · 1.17876 100 + 113 + 285 s13

This mean tetrachord of the first kind is generated by mean 13.

705 1.17867 · 1.06763 · 1.05956 285 + 113 + 100 s14

This mean tetrachord of the second kind is generated by mean 14.

706 1.17851 · 1.06771 · 1.05963 284 + 113 + 100 s15

This mean tetrachord of the second kind is generated by mean 17.

707 1.17851 · 1.06771 · 1.05963 282 + 114 + 101 s16

This mean tetrachord of the second kind is generated by mean 6.

708 $(4/3)^{1/5} \cdot (4/3)^{3/10} \cdot (4/3)^{1/2}$ 100 + 149 + 250 S17
This tetrachord is a semi-tempered interpretation of Aristoxenos's soft diatonic. In Cleonides's cipher, it is 6 + 9 + 15 parts.

709 1.07457 · 1.07457 · 1.154701 125 + 125 + 249 S18
This mean tetrachord of the first kind is generated by mean 2. The corresponding tetrachord of the second kind has the same intervals in reverse order.

710 $(4/3)^{2/15} \cdot (4/3)^{7/15} \cdot (4/3)^{2/5}$ 66 + 232 + 199 S19
This tetrachord is a semi-tempered interpretation of Aristoxenos's diatonic with soft chromatic diesis. In Cleonides's cipher, it is 4 + 14 + 12 parts.

711 1.13847 · 1.1250 · 1.0410 225 + 204 + 70 S20
This mean tetrachord of the third kind is produced by mean 5.

712 $(4/3)^{3/20} \cdot (4/3)^{9/20} \cdot (4/3)^{2/5}$ 75 + 224 +199 S21
This tetrachord is a semi-tempered interpretation of Aristoxenos's diatonic with hemiolic chromatic diesis. In Cleonides's cipher, it is 4.5 +13.5 + 12 parts.

713 1.13371 · 1.1250 · 1.04540 217 + 204 + 77 S22
This mean tetrachord of the third kind is produced by mean 14. In reverse order, it is generated by mean 13.

714 1.13315 · 1.1250 · 1.04595 216 + 204 + 78 S23
This mean tetrachord of the third kind is produced by the root mean square mean 17.

715 1.09185 · 1.07803 · 1.13278 152 + 130 + 216 S24
This mean tetrachord of the first kind is produced by mean 6.

716 1.09291 · 1.078328 · 1.13137 154 + 131 + 214 S25
This mean tetrachord of the first kind is produced by mean 17.

717 1.09301 · 1.07837 · 1.13122 154 + 131 + 213 S26
This mean tetrachord of the first kind is produced by mean 14. In reverse order is the tetrachord of the second kind generated by mean 13.

718 1.09429 · 1.07874 · 1.12950 156 + 131 + 211 S27
This mean tetrachord of the first kind is produced by mean 5.

719 1.12950 · 1.1250 · 1.04930 211 + 204 + 83 S28
This mean tetrachord of the third kind is produced by mean 6.

720 1.08866 · 1.1250 · 1.08866 147 + 204 + 147 S29
This mean tetrachord of the third kind is produced by the second or geometric mean.

721 $(4/3)^{1/5} \cdot (4/3)^{2/5} \cdot (4/3)^{2/5}$ 100 + 199 + 199 S30

This tetrachord is a semi-tempered interpretation of Aristoxenos's intense diatonic. In Cleonides's cipher, it is 6 + 12 + 12 parts.

722 $(4/3)^{1/3} \cdot (4/3)^{1/3} \cdot (4/3)^{1/3}$ 166 + 166 + 166 S31

Number 722 is the equally tempered division of the 4/3 into three parts. It is the semi-tempered form of Ptolemy's equable diatonic and of the Islamic neo-Aristoxenian approximation 10 + 10 + 10.

723 $(4/3)^{2/5} \cdot (4/3)^{3/10} \cdot (4/3)^{3/10}$ 200 + 149 + 149 S32

Number 723 is the semi-tempered version of the Islamic neo-Aristoxenian genus 12 + 9 + 9 parts.

Source index

The sources of the tetrachords listed below are the discoverers, when known, or the earliest reference known at the time of writing. Further scholarship may change some of these attributions. Because the Islamic writers invariably incorporated Ptolemy's tables into their compilations, they are credited with only their own tetrachords. The same criterion was applied to other historical works.

Permutations are not attributed separately except in notable cases such as that of Didymus's and Ptolemy's mutual use of forms of 16/15 · 9/8 · 10/9. Doubtful attributions are marked with a question mark.

For more information, including literature citations, one should refer to the entries in the Main Catalog. Uncredited tetrachords are those of the author.