Santa Fe Institute
2 talks on music and mathematics

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Talk 1:

- **Mathematics and Music (why?)**
- **Form and Distance**
- **Non-contextual analysis**

Talk 2:

- **Tuning**
- **Optimal Well-Temperament**

Mathematics and Music (why?) (Talk 1, Part 1)

- What is music? How can it be described? Why should it be described?
    - Some **examples** of diverse "styles" of music
        - What do these "pieces of music" have in common?
        - Style a set of rules, perhaps arbitrary, perhaps not. Historical, cognitive, cultural, political considerations.
        - Some of the most prominent questions that are asked about music are: What is cultural, what is innate? What is analyzable? What is overlearned?
        - What is music? A definition seems all but impossible. Some interesting postulates:
            - Varése "organized sound"
            - Cage: "work"
        - Music is *not* a language, though it is often described as such. Many theories of music assume things like *grammar* and syntax.
            - This is a common source of confusion. Music is sound without meaning.
                - You cannot tell someone to go to the corner and get a loaf of bread in music.
            - Syntax without semantics is not exactly, syntax. Grammar is more than a set of loose conventions for the order of events, or a (pedagogical) codification of a previous style.
            - Many attempts to analogize language and music, but often the lack of semantics are overlooked. The simplest linguistic utterance ("That's a cat!") is beyond the scope of music.

        - This distinguishes music from *all* other art forms. It is the only one incapable of representation (except insofar as it can represent its own history, as in, for example, tonal harmony).
        - Even though it's relationship to language is often confused, music is often venerated precisely for that reason: no meaning. It is regarded by other artists as a kind of "pure form." However, we still try to imbue various forms of meaning where there are none.
        - It is difficult, then, to "criticize" music, and teaching it becomes essentially the teaching of style, and the maintenance of a pedagogical tradition.
            - For example, many "justifications" of tonal music's "superiority," or "European music's superiority" seem to proceed from the assumption that they *are* generally superior because the explainer is most comfortable with them.
                - Not dissimilar to the overwhelming correlation of belief systems (religions) to those of one's family.
- Possible underlying, innate mechanisms? How do "humans" hear music?
    - One way to approach this would be to be as general as possible: assume more or less basic aspects of the way humans function in the world. Don't, in other words, start with the history of music.

- This does not by any means imply that all humans do these things in the same way, or hear things in the same way(s). Instead, we can consider *capabilities* and *propensities*.
- Some things that all humans seem to be able to do, aurally:
  - Parsing, ability to make distinctions.
    - Specifically: time-based distinctions.
  - Make judgments about similarity in several dimensional feature-spaces, and also, to utilize memory and prediction in doing so.
  - Distinguish pitch with varying levels of acuity.
    - Ability to perceive, discriminate and organize.
      - More specifically: to organize frequency into pitch in some way or ways.
    - Pitch has several dimensions: *height*, *equivalence class* (e.g. something like the octave, what Shepard, Tenney and others have called "chroma").
    - Pitch relationships general seem important in our aural experience, though the specific roles of contour and magnitude are still unclear.
  - Rhythmic acuity
    - Rhythm and time perception are even more multidimensional than pitch.
      - Some axes: pulse, duration, rhythmic complexity.
    - For example, a mathematical definition of *pulse* might be the "*mean*" duration, or more precisely, the mean delta start times between events
      - As a corollary, *rhythmic complexity* might be the *standard deviation*.

- Question: Is there some innate timer in the brain? And does that timer seem to prefer, or have an easier time with, or adjust to simple rational ratios. Does it also prefer, or perceive, or use in some way, some simple statistical mechanism for averaging delta start times (note: this seems like a good thing to do from an evolutionary perspective).

Music and mathematics (Talk 1, Part 2)

- Mathematics is one approach to musical description
- More or less context-, aesthetic-free. A possible way of describing musical processes, ideas, and more germane to my interests, possibilities.
- Without invoking culture, history of musical style, or agenda-ed aesthetic approaches (where, often, the conclusion precedes the argument)
- **On the other hand: Quote Xenakis**. The triviality of musico-mathematical problems seems, to me at least, a large and interesting problem, as well as an indictment of music (or perhaps the history of its explication).
- Why music and mathematics? What is there to solve?
- It is often said that there's a long and venerable history of the two, but I think that has become a kind of cliché (Pythagoras, Euler, Hofstadter, blah blah blah). They really have very little to do with each other historically. This is especially true in the modern era (say, post-Renaissance)
- There is a common interest by mathematicians in music, and vice versa. (Interestingly, musicians and mathematicians are said to be the people most adept at doing crosswords!). But.. there is a strange trend for scientists interested in music to be unusually conservative, even reactionary, about it (why not, as an obvious example, "Gödel, Escher, Schoenberg"?).
    - People want to "like" music, perhaps more so than other art. Unfamiliar music often makes people "physically angry," even violent. This doesn't seem to be the case, at least as much, with visual art, film, literature, or theatre. (Although the semantic content of those art forms has, in general, a much greater impact on people).
    - This may be linked to some kind of biological response to music that is not as salient in representational art forms. Perhaps, because of the complete lack of semantics, unusual music is truly incomprehensible, in a peculiarly frightening way.

- "Cross-disciplinary drift": it's nearly impossible to use one domain (cognition, mathematics, music history) to talk about music without invoking others as justifications. As interdisciplinarity, this is for the best.
    - But it's not always a good situation (even though nearly universal). One of the problems is that when talking about a different discipline, we don't often know what we're talking about.
        - For example: music theory books (counterpoint, harmony) very often begin with a "basic explanation of the physics of music" (which is, usually, completely wrong and uninformed). **EXAMPLE from *The Sacred Harp***.
- Music cognition, mathematics, history, convention, and something called "musical"
    - Most musical discussions tend to cross-disciplines cavalierly. Cognition, evolution, music history, style are lumped together, used as evidence for each other, as convenience. I will do this somewhat myself. I see no way around it. But…
    - We cannot, in general, answer deep musical questions definitively because we have no idea (or common agreement) on what music is, much less the question
    - One approach is to try and pose reasonable, well-defined, circumscribed questions, which perhaps invoke mathematical explanations. We hope that the mathematical explanations won't obscure the musical questions, but illuminate them, and in a sense, demystify them. Maybe the mathematics will even suggest new possibilities for music.
        - This is a very different act than trying to pose meaningful cognitive questions, in fact, much easier. Humans are messy. How many "possible" contours there are is an easier thing to get a handle on than how "humans hear contours."
    - I would like to believe, vis a vis the Xenakis quote, that the more interesting the mathematics involved, the deeper the musical question will tend to be. But there's no real evidence for this..
    - Example: *the contour problem* (Polansky and Bassein)

        - How many possible contours are there for musical melodies? (see below)
            - This is still baby mathematics, involving simple combinatorics.
        - The problem, though, was to state a problem generally, which had been "vaguely" implied by a lot of different work in music cognition, and music theory.
    - A *suspicion of naturalism*: a caveat about the use of cognition research in support of musical argument.
        - Naturalist arguments are as old as musical arguments, and many of them are, of course, important.
        - The "leap" from some cognitive, neuroscientific, psychological, or psychophysical mechanism, however, to a "musical conclusion" must be carefully inspected.
            - That there's evidence of an innate tendency has no direct bearing on "good" "bad," etc.
                - Stephen Pinker's famous example of our biological preferences fats, salts, and sugars: just because we have evolved to want them doesn't mean they're "good" for us.
                - *Xenophilia*: we are biologically programmed to be suspicious (and that's being nice) of anyone who looks different than us, but as a musical (and social) model, this seems like a bad idea. This is one instance where perhaps we might well choose to act contrary to our "programming."
                - Tendencies, are, after all, just that. Unusual and unfamiliar music is no more or less difficult for humans than runing a 10 second 100 meter dash, flying an airplane, planting a garden.
            - Cognitive and psycho-acoustic evidence is, in my opinion, widely abused in the discussion of musical fundamentals. An implicit assumption is that *tendency is equivalent to necessity*.

- Example: the magical number 7 as an explanation for the diatonic scale, simple experiments on "roughness" as a "justification" for tonality
- The "Mozart" effect (sometimes it's obvious fraud, sometimes it's less subtle)
- This is not a criticism of music cognition (nor of composers like James Tenney, Alvin Lucier, who do extraordinary and creative things with psycho-acoustical and cognitive phenomenon), which is crucially important to study. Rather: of it's applications.
  - Mathematicians are familiar with things like "math abuse."
- Lerdahl's article on "Cognitive Constraints…" is a good example of an attempt to connect diverse and specific music cognition examples to an overall aesthetic judgment (he quite clearly uses terms like "the best music", etc.)
- Maneris et. al also attempt to connect certain kinds of conformity with Zipf's law to "beautiful" in music (mainly, the value of the slope of a distribution).
- This is *not* something I (we) am trying to do.

▪ *Neophilia* one of the primary evolutionary traits
- Music has ALWAYS changed, moved "forward" (though that word has no meaning from an evolutionary standpoint). That's one of the strongest bits of cognitive evidence.
  - New music, new ideas seem like one of the primary traits argued by our brains, evolution.
  - Innovation, in some ways, seems to be true also in many animal "song" cultures, particularly songbirds.

▪ But the argument for neophilia as a justification for new music is no different than the argument for isolated cognitive mechanisms as a justification for the "status quo." Both turn a propensity into an aesthetic argument.

Form and Distance (Talk 1, Part 3)

- Hierarchical gestalt analysis
  - Tenney's *Meta + Hodos*
    - An attempt to formulate the understanding of contemporary music thru the model of Gestalt psychology.
    - Simple, "more or less" true observations on human perception used as a way of understanding music in time
    - A phenomenological approach.
  - Metric space model for hierarchical gestalt segmentation (Tenney and Polansky)
    - A computer model of this theory, on a highly circumscribed musical domain (monophonic music)
    - How to test it? A kind of pre-neural net technology neural-net approach (experienced humans as the evaluation function)?
  - Some **examples**: Varése *Density 21.5*; Ruggles *Portals*
- Morphology
  - Shape. Ordered values (a simple, general definition of morphology)
  - Contour, magnitude.
    - Contour requires the ability to judge whether something is "greater than," "less than," or the same as (in some general way).
    - Magnitude requires that we judge "how different" two things are.
      - Many people have trouble with musical magnitude (especially in pitch, especially "untrained" musicians). That is not the case for contour judgments, which seem much easier.

    - It's clear that both of these are perceptible, fundamental aspect of human perception. We can easily do both to some extent (in a wide variety of domains)
    - Great deal of psychoacoustic evidence that these two "features" operate at many levels, in relatively straightforward ways. Perhaps also independently, more than we think.
    - Contour perhaps more fundamental (example, we can all match contour, we have more trouble matching magnitude)
      - Difference between *linear* (is the first value bigger than the second?) and *combinatorial* (is the first value bigger than the third?) contour: Is the first value bigger than the third?
    - Still some open questions (How to order the musical contour space? How to discern, from the ternary contour descriptions, simply, which contour is possible, which "impossible").

- Distance and Metrics
  - What constitutes distances between musical morphologies?
    - This question can be posed in cognitive, musical, and mathematical terms.
  - A lot of standard mathematics deals with this (metric spaces, etc.)
  - Teitelbaum and Morris functions
    - Both from the literature of "atonal music theory," which has its own technical jargon, assumptions, etc.
    - These two functions, in some sense, transcend that literature and offer more general forms for measuring musical distance.
    - Teitelbaum's *Similarity Index* function is an early example of the use of an *L2 norm* in music, and a good way to compare multi-dimensional data.

        - Morris's *SIM* and *ASIM* functions are early examples of musical similarity functions on sets of *different cardinality.*
    - OCD metric (Polansky) as an example
        - Similar to the correlation function except that it does not return negative values.
        - Very detailed, and comes "close" to magnitude.
        - Has weights, various "knobs."
        - Seeks to find a kind of transparent criteria (contour) for similarity of morphologies.
    - Morphing: morphing vs. distance
        - Problem of morphing: distance of 1?
        - New ideas: Hausdorff distance?
- An example of metrics, multi-dimensional analysis: *The Casten Variation*
    - Based on an analysis of the Ruth Crawford Seeger *Piano Study in Mixed Accents* (1931).
    - An attempt to analyze an "unanalysable" piece, using different techniques, some standard, some not.
    - Led into multi-dimensional scaling of intra-morphological distances in the Seeger
    - Recreated some aspects of that distance matrix, then time-stretched and "range" stretched, to achieve an "analysis/resynthesis" of the original (thanks to Charles Dodge for that description).
    - New piece called *The Casten Variation*, a "reverse" engineering of MDS data, where the confusion matrix comes from an average of two morphological metrics.
    - Piece created in a non-deterministic fashion (at the time, I didn't know a simple way of doing this deterministically)

Non-contextual analysis (Talk 1, Part 4)

Introduction

- Christian Wolff: "we were all getting rid of the glue" (describing the early music of the "New York school")
- Like temporal gestalt analysis, an attempt at a stylistically-non-biased measurement
- In literature, can look at "glue" words. In art, frame statistics of visual data (with ever widening frames).
- Both of these have in common that the analysis procedure "knows nothing" about what it's trying to analyze. That's the virtue.
- What would that be in music? What are the musical glue words?

Some Recent Work (Polansky, Rockmore, Shapira, Haslett)

Introduction

- Maneris, McKay, others have recently worked on ways to try and delineate musical style by simple statistical analysis.
    - Zipf's law, MDS, PCA seem to be common techniques, ideas.
    - Compression analysis another promising avenue for future work.
- There is a long history of this kind of work, dating back, in modern times, to Hiller's work on stochastic simulations of computer styles (like the *Illiac Suite* for string quartet, from the 1950s).
- Most of the work that has been done has been rule-based, and if non-rule based, a more or less non-deterministic approach to simulating musical rules (thus style).
- NCA is a different approach to the problem of style identification (and thus, historical musical analysis). It purports to know "as little as possible" about the artistic styles and conventions it is addressing.
- It tends to look for "signatures" that might not be thought salient in standard, intra-disciplinary approaches.

- For example, looking for musical motives, or key structures, or melodic patterns might be too high a level for this sort of work.

A description of the current state of our work (Polansky, Rockmore, Shapira, Haslett)

Tuning (Talk 2, Part 1)

- The fundamentals of tuning and the *historical tuning problem*
    - Not a problem to mathematicians (that is, the fundamental theorem of arithmetic is not viewed as being something that needs to be solved!)
    - But in music, since primes are the basic building blocks of tuning, this is primarily a practical question because of *constraints* like: *fixed number of notes*, *fixed number of "keys"*, *strings*, etc.
        - These constraints need to be resolved against certain *idealizations of tuning* (simple rational ratios, the ability to play in "different keys" or "modes"; relative importance of certain intervals)
    - Interestingly, not really a problem in choirs, string quartets, or variable pitch instruments (like computers) but they use the same paradigm, since their larger world is "fixed pitch".
    - There are few, if any, large musical cultures, which don't have some notion of fixed pitch, as well as some notion of ideal intervals. However, these are expressed in widely different ways, both over geography, and over time
        - Example: description of pelog as a subset of 9-ET. A "traditional" system recast in the language of western European art music language. (ADD GRAPHIC HERE).
    - The history of descriptions of tuning in western music is long and diverse.
- Harmonic complexity and distance (how to measure something called "consonance"?)
    - Euler GS function, Tenney's HD function
- The historical development of the solution to the historical tuning problem
    - The wolf fifth, and the generalization (the "canidae" problem)

- Microtonalism…? Development of "alternate" scales, tuning systems, notations, theories
- I prefer to call it *experimental intonation*, or just "tuning"
  - **Example**: *Collaruncincla*…

OWT (Optimal Well Temperament) (Polansky, Rockmore, Johnson, Repetto) (Talk 2, Part 2)

- What is well-temperament?
  - A solution to multiple keys, multiple intervals, fixed number of scale degrees.
  - Nicely viewed as a matrix.
- **Examples** of Bach, WTC, two different pieces, two different tunings
- Show the optimization algorithm, explain it as a constraint problem, solving the "historical tuning problem."
- Talk about the problem of slendro: five tone scale, several modes, and stretched octave.
- Genetic and deterministic approaches: run the JAVA code, and the Matlab code.
  - JAVA program to solve the problem as a genetic algorithm.
  - Ends up giving exactly the same answers (so far) as the deterministic solution.
- Constraints and error.
  - Fixed number of notes, ideal intervals, key and interval weights, taking the error function on the matrix.
- How does equal temperament evolve?
  - The "whatever" solution.
- Going the other direction: giving an historically important scale: finding the constraints?
  - More than one solution possible.
  - Does this algorithm need to be proved mathematically?
  - What are the mathematical implications of this algorithm?
  - Strange artifacts of the system:
    - bad scales (related to music theoretical notions of *proprietary* scales)
    - equal temperament as a kind of default condition

            - multiple solutions (changing key or interval weights can have the same affect).
            - equal temperament emerging as the "average" of all scale intervals, because of the "wrap around" formation of the matrix.
    - Proof of concept
        - Some simple scales: first with some simple constraints (a set of just intervals, equal-temperament).
        - Preexisting scales: how to generate them via ideal intervals e.g W3, slendro).
        - Explain simple algorithm for fixing ideal intervals, searching for weights.
            - Have to fix something or the search space becomes too big.
            - Not too hard to find very good W3 and slendro
                - Big surprise: Intervals used to find slendro conform to some of my ideas (and some of Lou Harrison's) from 1984s about the genesis of slendro (*Balungan*). In particular, making the 2nd scale degree an 8/7 and then "tempering" the fifths about half of the octave "stretch" creates an exact fit to this particular slendro.
                - But… there are a *lot* of slendros!
    - What would it mean for one of these to sound good? What does it mean for any scale to sound good?
    - "bad scales" (improper, strictly improper, "extremely" improper) when the constraints get too constrain-y.
    - Hope: the end of scales

Further Readings
Polansky, Two Lectures,
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