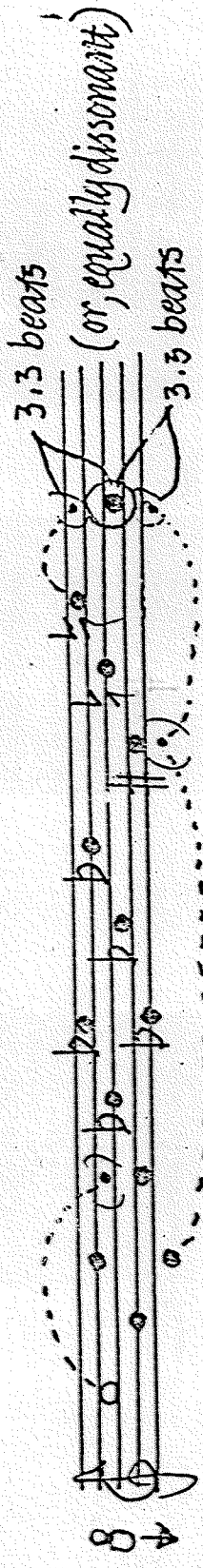


Johann Philipp Kirnberger, 1779, $\frac{1}{2}$ syntonic comma Well Temperament

HOW TO TUNE



(all without beats save last tone)

FOR A METER STRING: $\frac{1000}{949}$ $\frac{900}{843.75}$ $\frac{800}{750}$ $\frac{711.91}{670.82}$ $\frac{632.81}{600}$ $\frac{562.5}{533}$ $\frac{94}{500}$

Alternate Tunings and Scales

Werckmeister III

Andreas Werckmeister was a musical theorist of Bach's day who was concerned with adjusting the tuning of keyboard instruments so that they could be played in any key. It was under this impetus that Bach wrote the famous *Well-Tempered Clavier*. Contrary to popular belief, this set of pieces written in each of the major and minor keys was not intended for performance in equal temperament. Performed in a temperament such as Werckmeister's, each key exhibits a unique character which Bach himself described in his writings. This is one of the most popular temperaments devised by Werckmeister.

Scale Degree	Key Map	Interval to 1st Degree		Consecutive Intervals	
		Ratio	Cents	Ratio	Cents
1	C1	1.00	0.00		
2	C#1	1.06	93.67	1.06	93.67
3	D1	1.12	192.18	1.06	98.51
4	D#1	1.17	277.69	1.05	85.51
5	E1	1.25	390.23	1.07	112.54
6	F1	1.32	474.58	1.05	84.36
7	F#1	1.40	588.27	1.07	113.69
8	G1	1.49	696.10	1.06	107.83
9	G#1	1.58	790.23	1.06	94.14
10	A1	1.67	888.28	1.06	98.05
11	A#1	1.76	974.79	1.05	86.52
12	B1	1.88	1092.19	1.07	117.40
13	C2	2.00	1200.00	1.06	107.82

Vallotti and Young

Francescantonio Vallotti and Thomas Young independently devised an adjustment to the Pythagorean tuning in which the first six fifths are lowered by 1/6 of a Pythagorean comma. This temperament, derived from a key-based tuning, is capable of performance in any key.

Scale Degree	Key Map	Interval to 1st Degree		Consecutive Intervals	
		Ratio	Cents	Ratio	Cents
1	C1	1.00	0.00		
2	C#1	1.06	93.67	1.06	93.67
3	D1	1.12	196.09	1.06	102.42
4	D#1	1.17	277.69	1.05	81.60
5	E1	1.25	392.18	1.07	114.49
6	F1	1.32	474.58	1.05	82.40
7	F#1	1.40	588.27	1.07	113.69
8	G1	1.50	698.05	1.07	109.78
9	G#1	1.58	790.23	1.05	92.18
10	A1	1.68	894.14	1.06	103.91

Table II. Chromatic scale of 'Werckmeister III'. The unit of deviation is the quarter of a comma. The column with string lengths is after Türk (1808, pp.477-482).

Tone	Deviation from Werckmeister I	Relative pitch (cents)	Türk's string lengths	Idem, converted into cents
C	C 0	0	8192	0
Csharp	Cis +4	90.226	7776	90.225
D	D -2	192.180	7331	192.247
Eflat	Dis +4	294.135	6912	294.135
E	E +1	390.225	6540	389.910
F	F 0	498.045	6144	498.045
Fsharp	Fis 0	588.270	5832	588.270
G	G -1	696.090	5480	696.048
Gsharp	Gis +4	792.181	5184	792.180
A	A +1	888.270	4905	887.955
Bflat	As +4	996.090	4608	996.090
B	H +1	1092.180	4360	1091.865
c		1200	4096	1200
unequality inaccuracy		6.484		0.139

Table III. Temperings of consonant intervals and mean temperings of triads and keys in 'Werckmeister III'. The unit of tempering of the fifth and the major thirds is one quarter of a neutral comma (about 5.5 cents), as used by Werckmeister. D=ditonic comma.

Fundamental tone	tempering of						mean tempering	
	fifth			major third		minor third	major triad	key
	units	commas	cents	units	cents			
Eflat	0	0	0	3	15.641	-21.506	10.428	10.265
Bflat	0	0	0	2	9.776	-21.506	6.518	9.613
F	0	0	0	1	3.912	-21.506	2.608	6.124
C	-1	-D	-5.865	1	3.912	-21.506	6.518	9.124
G	-1	-D	-5.865	2	9.776	-15.641	10.428	9.450
D	-1	-D	-5.865	2	9.776	-9.776	10.428	9.939
A	0	0	0	3	15.641	-3.912	10.428	10.591
E	0	0	0	3	15.641	-9.776	10.428	11.242
B	-1	-D	-5.865	3	15.641	-15.641	14.338	11.731
Fsharp	0	0	0	4	21.506	-15.641	14.338	11.731
Csharp	0	0	0	4	21.506	-15.641	14.338	11.405
Gsharp	0	0	0	4	21.506	-15.641	14.338	10.916
mean (of tuning)							10.428	10.428

Table IV. Frequencies of tones, and fourths, and major thirds of the chromatic scale of 'Werckmeister III', calculated

Fundamental tone	Frequency (Hz)	Fifth
C	131.702	-1.3
Csharp	138.748	0
D	147.164	-1.4
Eflat	156.091	0
E	165	0
F	175.603	0
Fsharp	184.997	0
G	196.885	-1.9
Gsharp	208.122	0
A	220	0
Bflat	234.137	0
B	247.5	-2.5
middle C	263.404	-2.6
Csharp	277.496	0
D	294.329	-2.9
Eflat	312.183	0
E	330	0
F	351.206	0
Fsharp	369.994	-
G	393.770	-
Gsharp	416.244	-
A	440	-
Bflat	468.274	-
B	495	-
C	526.808	-

Table II. Chromatic scale of 'Werckmeister III'. The unit of deviation is the quarter of a comma. The column with string lengths is after Türk (1808, pp.477-482).

Tone	Deviation from Werckmeister I	Relative pitch (cents)	Türk's string lengths	Idem, converted into cents
C	C 0	0	8192	0
Csharp	Cis +4	90.226	7776	90.225
D	D -2	192.180	7331	192.247
Eflat	Dis +4	294.135	6912	294.135
E	E +1	390.225	6540	389.910
F	F 0	498.045	6144	498.045
Fsharp	Fis 0	588.270	5832	588.270
G	G -1	696.090	5480	696.048
Gsharp	Gis +4	792.181	5184	792.180
A	A +1	888.270	4905	887.955
Bflat	As +4	996.090	4608	996.090
B	H +1	1092.180	4360	1091.865
c		1200	4096	1200
unequality inaccuracy		6.484		0.139

Table III. Temperings of consonant intervals and mean temperings of triads and keys in 'Werckmeister III'. The unit of tempering of the fifth and the major thirds is one quarter of a neutral comma (about 5.5 cents), as used by Werckmeister. D=ditonic comma.

Fundamental tone	tempering of						mean tempering	
	fifth			major third		minor third cents	major triad cents	key cents
	units	commas	cents	units	cents			
Eflat	0	0	0	3	15.641	-21.506	10.428	10.265
Bflat	0	0	0	2	9.776	-21.506	6.518	9.613
F	0	0	0	1	3.912	-21.506	2.608	6.124
C	-1	-1/2 D	-5.865	1	3.912	-21.506	6.518	9.124
G	-1	-1/2 D	-5.865	2	9.776	-15.641	10.428	9.450
D	-1	-1/2 D	-5.865	2	9.776	-9.776	10.428	9.939
A	0	0	0	3	15.641	-3.912	10.428	10.591
E	0	0	0	3	15.641	-9.776	10.428	11.242
B	-1	-1/2 D	-5.865	3	15.641	-15.641	14.338	11.731
Fsharp	0	0	0	4	21.506	-15.641	14.338	11.731
Csharp	0	0	0	4	21.506	-15.641	14.338	11.405
Gsharp	0	0	0	4	21.506	-15.641	14.338	10.916
mean (of tuning)							10.428	10.428

Table IV. Frequencies of tones, and beat fourths, and major thirds of the octave of 'Werckmeister III', calculated for a

Fundamental tone	Frequency (Hz)	Beat fifth
C	131.702	-1.336
Csharp	138.748	0
D	147.164	-1.493
Eflat	156.091	0
E	165	0
F	175.603	0
Fsharp	184.997	0
G	196.885	-1.998
Gsharp	208.122	0
A	220	0
Bflat	234.137	0
B	247.5	-2.511
middle C	263.404	-2.672
Csharp	277.496	0
D	294.329	-2.986
Eflat	312.183	0
E	330	0
F	351.206	0
Fsharp	369.994	-
G	393.770	-
Gsharp	416.244	-
A	440	-
Bflat	468.274	-
B	495	-
C	526.808	-

er III'. The unit of deviation with string lengths is after

Türk's string lengths	Idem, converted into cents
8192	0
7776	90.225
7331	192.247
6912	294.135
6540	389.910
6144	498.045
5832	588.270
5480	696.048
5184	792.180
4905	887.955
4608	996.090
4360	1091.865
4096	1200
	0.139

ervals and mean temperings of. The unit of tempering of the rter of a neutral comma (about =ditonic comma.

third cents	minor third cents	mean tempering	
		major triad cents	key cents
10.641	-21.506	10.428	10.265
9.776	-21.506	6.518	9.613
8.912	-21.506	2.608	6.124
8.912	-21.506	6.518	9.124
9.776	-15.641	10.428	9.450
9.776	-9.776	10.428	9.939
5.641	-3.912	10.428	10.591
5.641	-9.776	10.428	11.242
5.641	-15.641	14.338	11.731
11.506	-15.641	14.338	11.731
11.506	-15.641	14.338	11.405
11.506	-15.641	14.338	10.916
		10.428	10.428

Table IV. Frequencies of tones, and beat frequencies of fifths, fourths, and major thirds of the octave below and above middle C, of 'Werckmeister III', calculated for a'=440 Hz.

Fundamental tone	Frequency (Hz)	Beat frequency (Hz) of		
		fifth	fourth	major third
C	131.702	-1.336	0	1.490
Csharp	138.748	0	0	8.672
D	147.164	-1.493	1.998	4.167
Eflat	156.091	0	0	7.083
E	165	0	0	7.488
F	175.603	0	0	1.986
Fsharp	184.997	0	2.511	11.562
G	196.885	-1.998	2.673	5.575
Gsharp	208.122	0	0	13.007
A	220	0	2.986	9.983
Bflat	234.137	0	0	6.630
B	247.5	-2.511	0	11.231
middle C	263.404	-2.672	0	2.979
Csharp	277.496	0	0	17.343
D	294.329	-2.986	3.995	8.334
Eflat	312.183	0	0	14.166
E	330	0	0	14.975
F	351.206	0	0	3.972
Fsharp	369.994	-	5.022	23.124
G	393.770	-	5.345	11.150
Gsharp	416.244	-	-	26.015
A	440	-	-	-
Bflat	468.274	-	-	-
B	495	-	-	-
C	526.808	-	-	-