Melodic Transformations

("8th of January")

OCD (Ordered Combinatorial Direction)

$$\frac{\sum_{i=1}^{L-1} \sum_{j=i+1}^{L} diff\left(sgn\left(\Delta(M_i, M_j)\right), sgn\left(\Delta(N_i, N_j)\right)\right)}{L_m} \qquad (OCD)$$

The *OCD* is the combinatorial version of the *OLD*. It is the most discriminating of the direction metrics. The *OCD* measures the complete, cell-by-cell network of contour similarity between two morphs.The *OCD* closely reflects melodic perception, tracking the difference between the combinatorial contour of two melodies.[48]

The *OLD, ULD* and *OCD* are generally independent (though the *OLD* is "included" as the diagonal for the *OCD*). For the two morphs (above) `M={5,9,3,2}, N={2,5,6,6}` with combinatorial contour matrices:[49]

```
-  +  +           -  -  -
   +  +    and       -  -
      +                 0
```

— the *diff* value between corresponding cells is 5, so $OCD(M,N) = .83$ [50]. If `O={5,3,6,1,4}` and `P={3,6,1,4,2}` (see the above comparison of the*ULD* and *OLD*), with contour matrices:

```
+  -  +  +           -  +  -  +
   -  +  -    and       +  +  +
      +  +                 -  -
         -                    +
```

— the *diff* value is 8, so $OCD(O,P) = .8$ (again, very different), where the $OLD(O,P) = 1$ (completely different "diagonals") and $ULD(O,P) = 0$ (half up, half down for each).

The *OCD*'s values range from [0,1] with a grain of $1/L_m$.[51]

UCD (Unordered Combinatorial Direction)

VERY INCOMPLETE, IN PROGRESS!!!!!!!!!!!

$$\left| \frac{\sum_{i=2}^{L} \Delta(e_{1_i}, e_{1_{i-1}})}{L-1} - \frac{\sum_{i=2}^{L} \Delta(e_{2_i}, e_{2_{i-1}})}{L-1} \right|$$

Comments: Measures the difference between the average linear interval in each morphology. That is, this compares the average "width" of interval in one morphology to that of the second morphology, but does not preserve where that "width" came from within each morphology.

Unordered Linear Absolute Magnitude (ULAM)

$$\sum_{i=1}^{L} \frac{\Delta(e_{1_i}, e_{2_i})}{L}$$

Comments: A much more primitive version of ULM. In fact, to understand the ULAM, ULM might be thought of as ULIM — Unordered Linear Intervalic Magnitude. The ULM is simply the first order difference function of the ULAM. An important thing to note is that the ULAM is *not necessarily a metric.* Since Δ is not necessarily an unsigned value, the ULAM may return a negative value (thanks to Nick Didkovsky for pointing this out). It can be made into a true metric by the insertion of an absolute value sign, and the careful choice of the Δ. This "metric" is simply an interpolation function.

Oordered Combinatorial Magnitude (OCM)

$$\frac{\sum_{j=1}^{L-1} \sum_{i=1}^{L-j} \left| \Delta(e_{1_i}, e_{1_{i+j}}), - \Delta(e_{2_i}, e_{2_{i+j}}) \right|}{L_m}$$

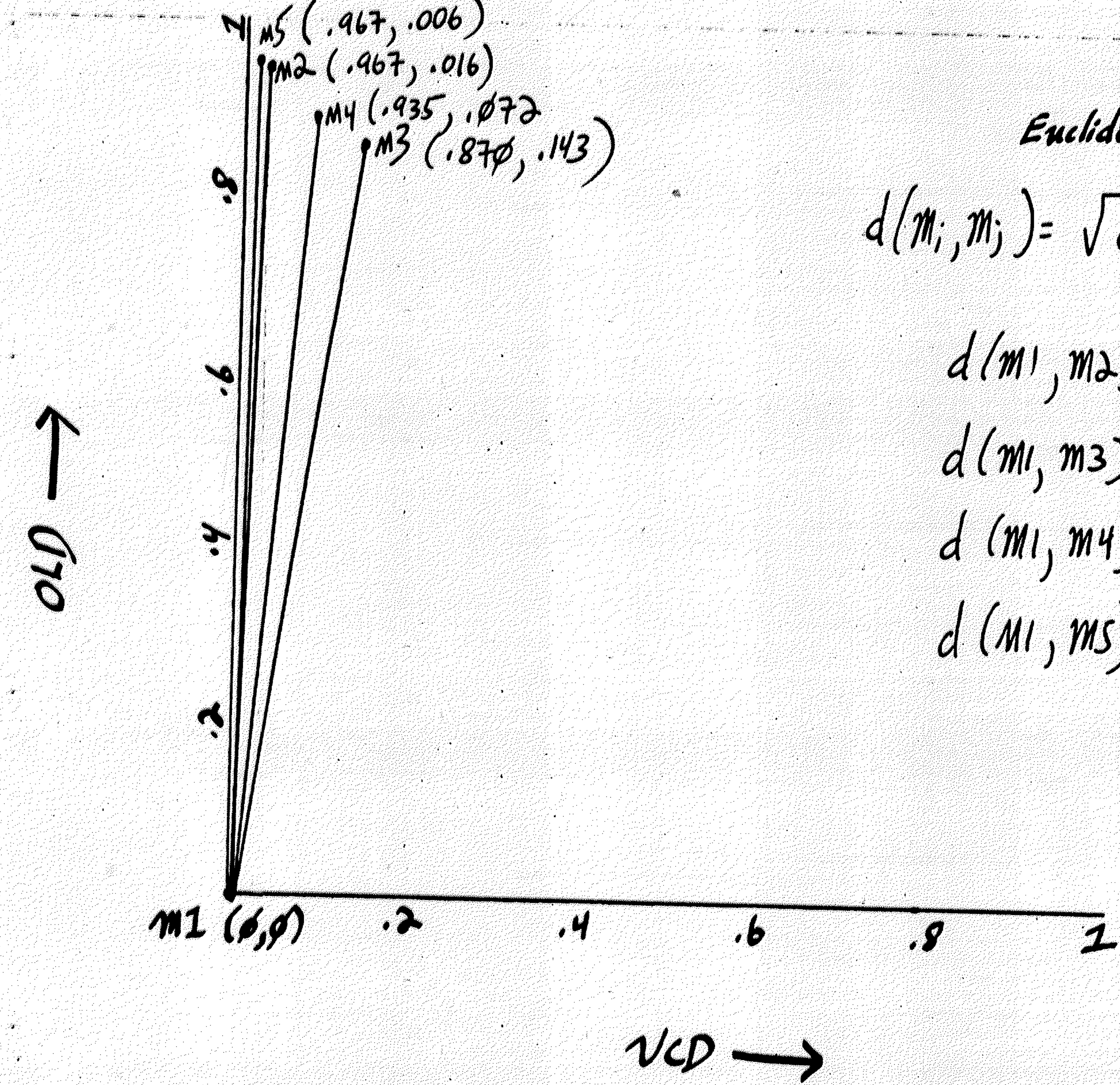
Comments: A combinatorial version of the OLM, but note that the OCM is redundant if weights or weighting functions are not added to individual rows and columns. That is, a combinatorial version of linear magnitudes gives not more significant information than a linear one, since the combinatorial magnitudes can be "inferred" from the linear ones. However, if weighting functions are added, this becomes a truly significant measure, and far more sensitive than the OLM.

Unordered Combinatorial Magnitude (UCM)

$$\left| \frac{(\sum_{k=1}^{L-1} (\sum_{i=1}^{L-j} \Delta(e_{1_i}, e_{1_{i+j}})))}{L_m} - \frac{(\sum_{k=1}^{L-1} (\sum_{i=1}^{L-j} \Delta(e_{2_i}, e_{2_{i+j}})))}{L_m} \right|$$

Comments: Even less useful than OCM if weights are not used.

Distances in two-dimensional metric space from m1



Euclidean metric:

$$d(m_i, m_j) = \sqrt{OLD(m_i, m_j)^2 + NCD(m_i, m_j)^2}$$

$d(m1, m2) = .9670186$

$d(m1, m3) = .9671323$

$d(m1, m4) = .937768$

$d(m1, m5) = .88167$