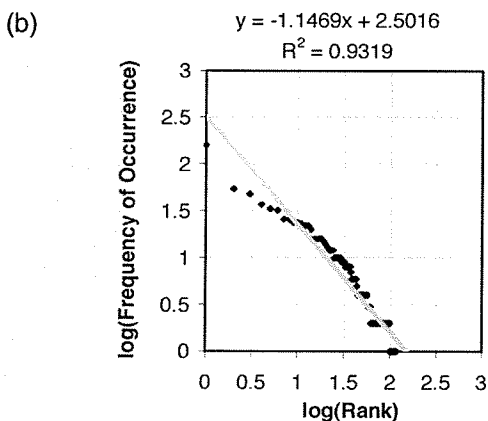
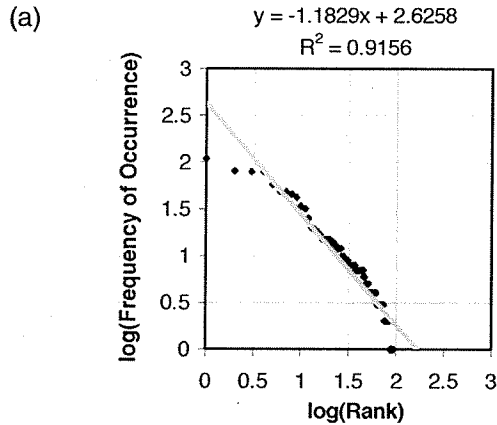


Figure 1. (a) Rank-frequency distribution of melodic intervals for Chopin's Revolutionary Etude, Op. 10 No. 12 in C minor; (b) rank-frequency

distribution of chromatic-tone distance for Bach's Orchestral Suite No. 3 in D, movement no. 2, Air on the G String, BWV 1068.



Mozart's *Bassoon Concerto in B-flat Major* is inversely related to the length of these time intervals (Zipf 1949, p. 337). In other words, plotting the counts of various distances against the actual distances (from smaller to larger) produces a near-Zipfian line.

Using size instead of rank on the x-axis generates a *size-frequency* distribution. This is an alternative formulation of Zipf's Law that has found application in architecture and urban studies (Salingaros and West 1999). This formulation is also used in the *box-counting* technique for calculating the fractal dimension of phenomena (Schroeder 1991, p. 214).

Zipf's Law has been criticized on the grounds that $1/f$ noise can be generated from random statistical processes (Li 1992, 1998; Wolfram 2002, p. 1014). However, when studied in depth, one realizes that Zipf's Law captures the scaling properties of a phenomenon (Mandelbrot 1977, p. 345; Ferrer Cancho

and Solé 2003). In particular, Benoit Mandelbrot, an early critic, was inspired by Zipf's Law and went on to develop the field of fractals. He states:

Natural scientists recognize in "Zipf's Laws" the counterparts of the scaling laws which physics and astronomy accept with no extraordinary emotion—when evidence points out their validity. Therefore physicists would find it hard to imagine the fierceness of the opposition when Zipf—and Pareto before him—followed the same procedure, with the same outcome, in the social sciences. (Mandelbrot 1977, pp. 403–404)

Zipf-Mandelbrot Law

Mandelbrot generalized Zipf's Law as follows:

$$P(f) \sim 1/(1 + b)^{f^{1+c}} \quad (3)$$

where b and c are arbitrary real constants. This is known as the *Zipf-Mandelbrot Law*. It accounts for natural phenomena whose scaling properties are not necessarily Zipfian.

Zipf's Law in Music

Zipf himself reported several examples of $1/f$ distributions in music. His examples were processed manually, because computers were not yet available. Zipf's corpus consisted of Mozart's *Bassoon Concerto in B-flat*; Chopin's *Etude in F minor, Op. 25, No. 2*; Irving Berlin's *Doing What Comes Naturally*; and Jerome Kern's *Who*. This study focused on melodic intervals and the distance between repetitions of notes (Zipf 1949, pp. 336–337).

Richard Voss and John Clarke (1975, 1978) conducted a large-scale study of music from classical, jazz, blues, and rock radio stations recorded continuously over 24 hours. They measured several parameters, including output voltage of an audio amplifier, loudness fluctuations of music, and pitch fluctuations of music. They discovered that pitch and loudness fluctuations in music follow Zipf's distribution. Additionally, Voss and Clarke developed a computer program to generate music using three different ran-