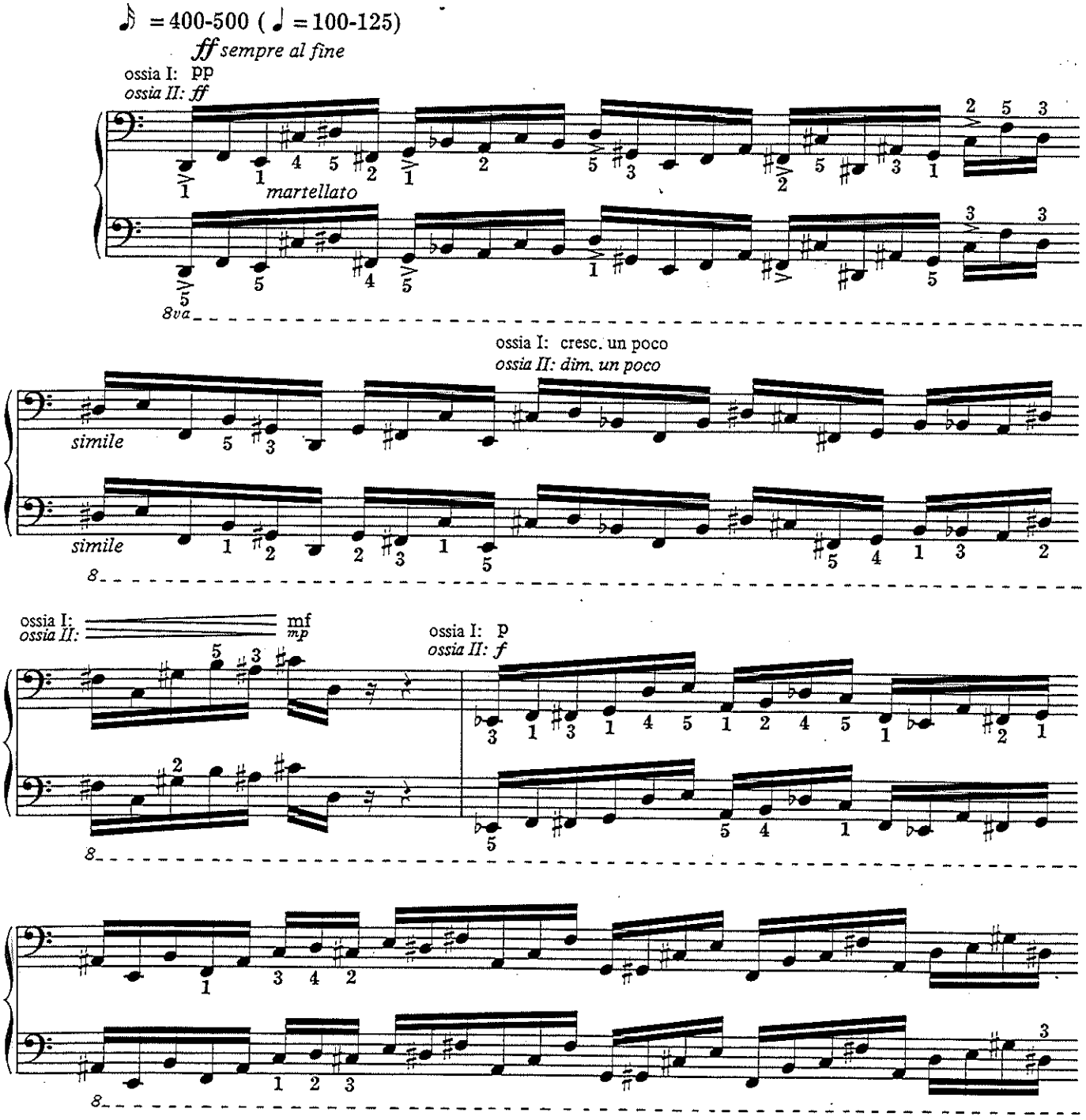
Piano Study
in Mixed Accents

Accidentals affect only the immediate note.
The first note of each group of sixteenths must be strongly accented. It should be played in a tempo that allows the accents to be clearly brought out.
One of the three dynamic possibilities (ff sempre al fine, ossia I, ossia II) should be adhered to throughout a performance.

RUTH CRAWFORD



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Atonal analysis

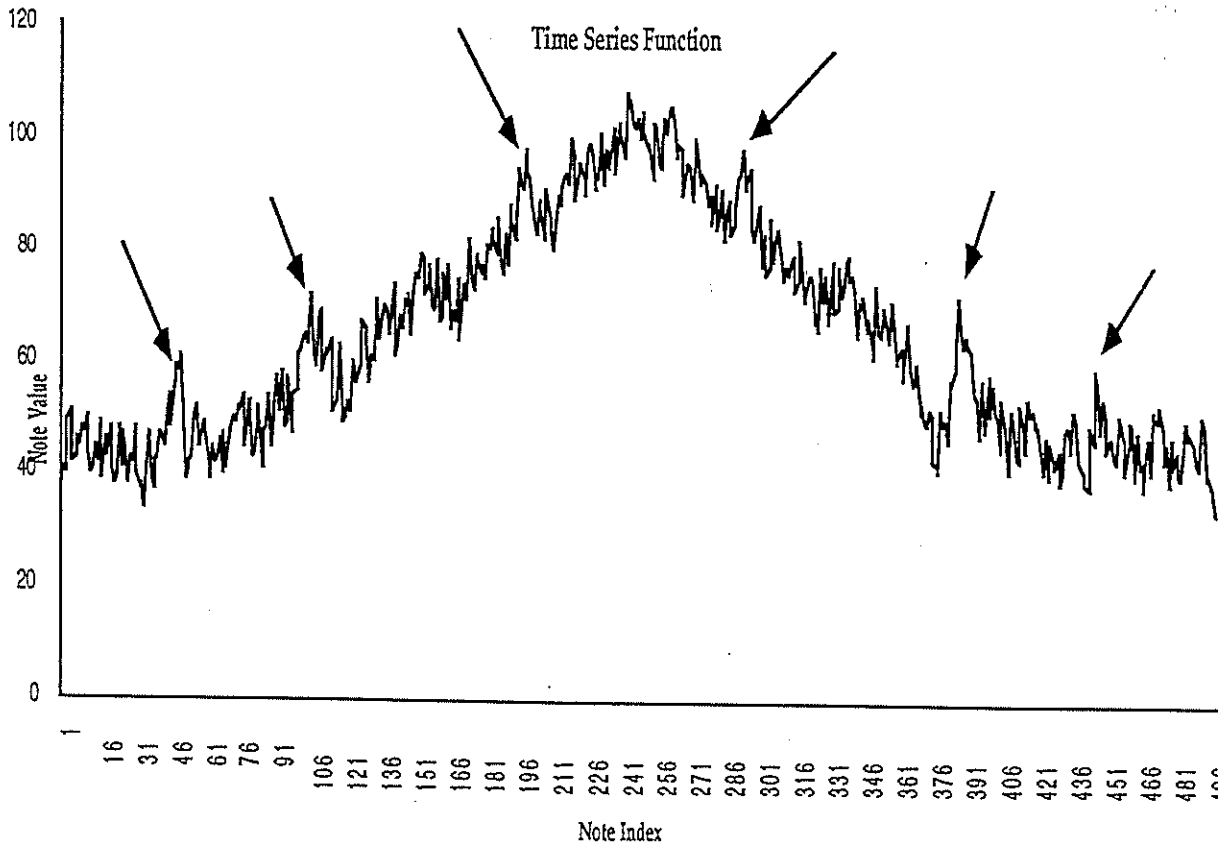
The following table is an excerpt of a set-class analysis of the beamed rhythmic groupings in Ruth Crawford's *Study in Mixed Accents* (1930) for solo piano. It was done by the class members of Music 23, Advanced Music Theory, Dartmouth College, Spring 93, as an example project. (Courtney Kennedy, Eric Ochnsner, David Sullivan, Brandon Adams, Ted Coffey and myself) The chart is grouped vertically by "measures," sets of beamed groupings separated by pauses, of which there are only five in the score. These "sections" articulate the mirror image form of the piece.

Key

#asc.: number of the beamed grouping, starting at the first.
#desc.: number of the beamed grouping starting at the last (there are 111 groupings). This number is used because of the mirror/retrograde structure of the work.
p/l: page and line#'s of the grouping in the standard edition of the score from Presser. For example, 2-1 means second page, 1st line.
notes: pitch classes in beamed grouping, using *A* for 10 and *B* for 11.
int.: successive intervals of the grouping, reduced to within the octave but not collapsed around the inversion around the tritone. In other words, major sevenths (11) are not equal to minor seconds (1).
red. int.: successive intervals of the grouping, reduced to within the octave and collapsed to the tritone inversion. In other words, 6 = tritone, 3 = minor third *and* major sixth, and so on.
normal: normal order of the beam grouping. This normal order is of the grouping itself, and is not necessarily the "best normal order" between the grouping and its inversion. In other words, it is the minimal right-most span of the pitch classes within one octave.
prime: the conventional Forte prime-form pitch-class set for the pitch classes in the grouping.
Forte #: the "name" of the pc-set (prime-form), and the one found in standard pc-set-class tables.
IV: standard interval vector for the Fort pc-set class.

#asc	#desc	p/l	notes	int.	red. int.	normal	prime	Forte #	IV
(Section 1)									
1	111	2-1	254136	319B9	31313	123456	012345	6-1	543210
2	110	2-1	7A91B	3131	3131	79AB1	02346	5-8	232201
3	109	2-1	28457	6414	6414	23478	01346	5-10	223111
4	108	2-1	623A7	7B71	5151	2376A	01458	5-21	202420
5	107	2-1	152	53	53	125	014	3-3	101100
6	106	2-2	345B82	18646	14646	23458B	012369	6-z42	324222
7	105	2-2	7614	168	164	1467	0136	4-13	112011
8	104	2-2	23A5B	1455	1455	AB235	01457	5-z18	212221
9	103	2-2	3267	271	251	2367	0145	4-7	201210
10	102	2-2	BA92	116	116	9AB2	0125	4-4	211110
11	101	2-3	608BA	6831	6431	068AB	01246	5-9	231211
12	100	2-3	12	---	---	---	---	---	---
(Section 2)									
13	99	2-3	356724	21172	21152	234567	012345	6-1	543210
14	98	2-3	9B10	221	221	9B01	0124	4-2	221100
15	97	2-3	54967	2631	2631	3567B	01248	5-13	221311
16	96	2-3	A4B59	6764	6564	459AB	01267	5-7	310132
17	95	2-4	021	21	21	012	012	3-1	210000
18	94	2-4	436905	13935	13635	034569	012369	6-z42	324222
19	93	2-4	7814	153	153	1478	0147	4-18	102111
20	92	2-4	5B069	6169	6163	569B1	01468	5-30	121311
21	91	2-4	2381	245	245	2348	0126	4-5	210111
22	90	3-1	A01	319	316	A01	013	3-2	111000
23	89	3-1	9B	---	---	---	---	---	---
24	88	3-1	6712	1514	1514	1267	0156	4-8	200121
25	87	3-1	45160	1A364	12364	03456	01236	5-4	322111
26	86	3-1	1B89A	A9B3	2313	89AB1	01235	5-2	432100
27	85	3-1	2145673	B31118	131114	1234567	0123456	7-1	654321

Piano Study... as time series (showing "periodicity ripples")



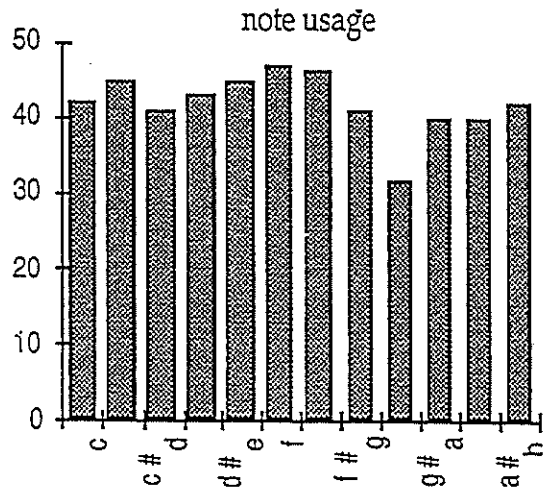
General Statistics of Pitch

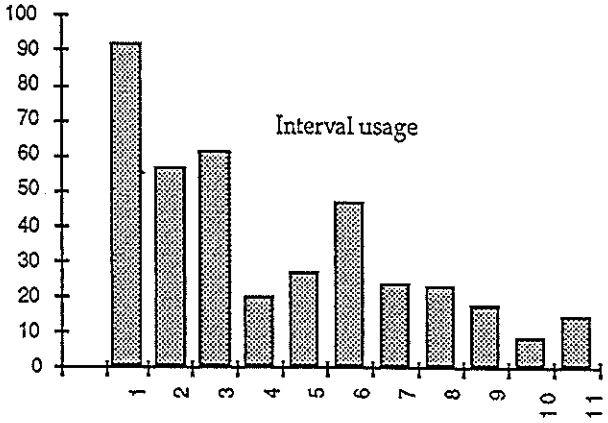
Total number of notes $L = 504$
Mean pitch $\mu = 64.56$ (app. E above middle C)
Mean interval (non-tritone reduced) $\mu = 4.35$
Mean interval (tritone reduced) $\mu = 2.99$

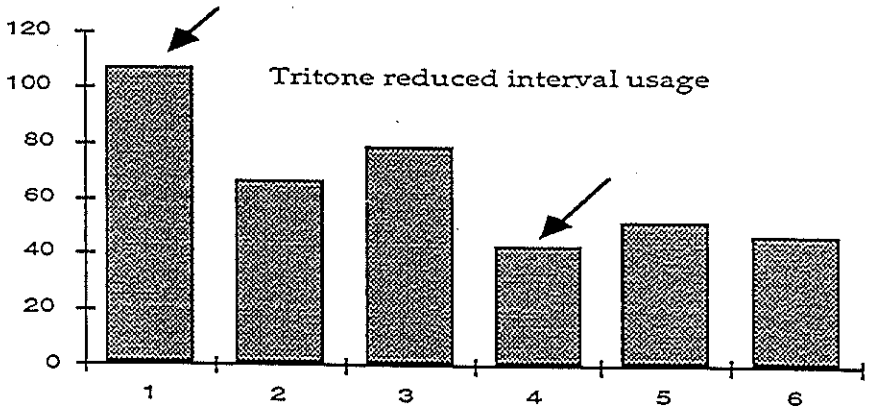
Standard deviation of pitch $\delta = 19.468$
St. dev. of (non-tritone reduced) interval $\delta = 3.066$
St. dev. of (tritone-reduced) interval $\delta = 1.698$

High pitch of piece is: 108 (D)
Low pitch of piece is: 34 (Bb)
Range of piece is: 74 (6 octaves and a major second)

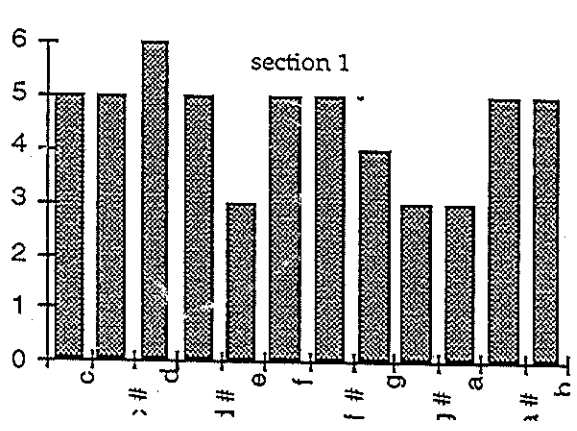
Note and interval histograms, entire piece

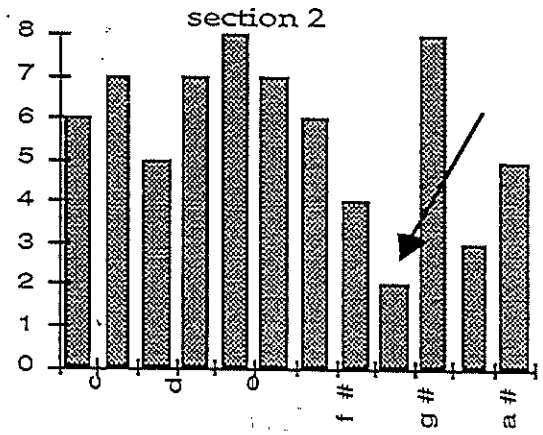




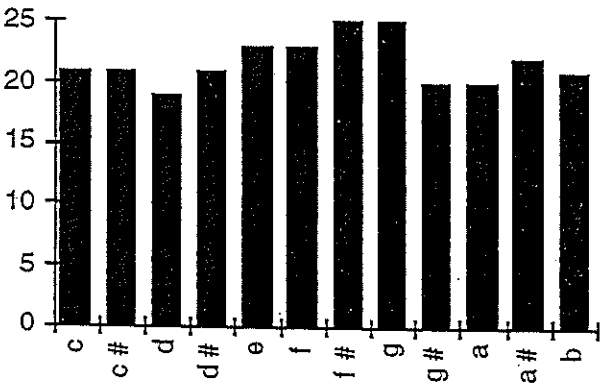


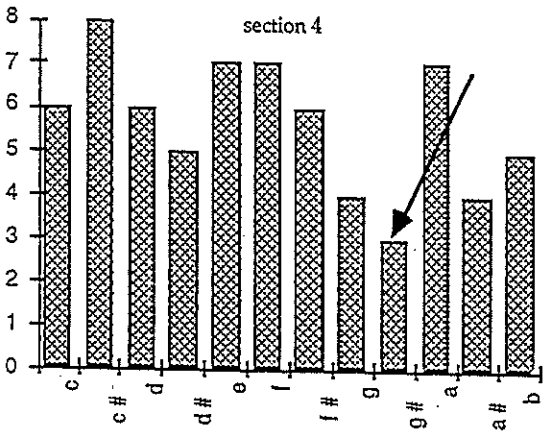
Note histograms, mod 12, all sections

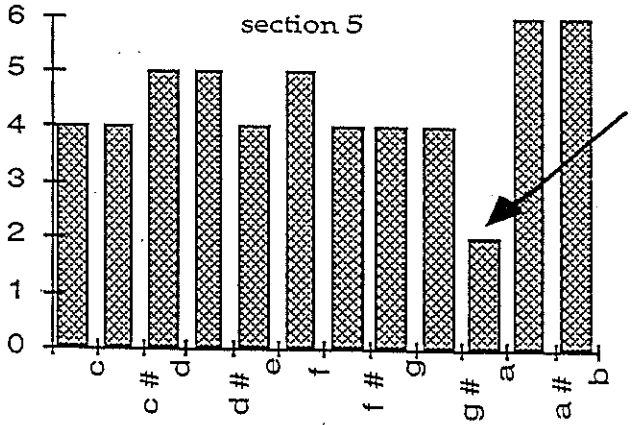




section 3

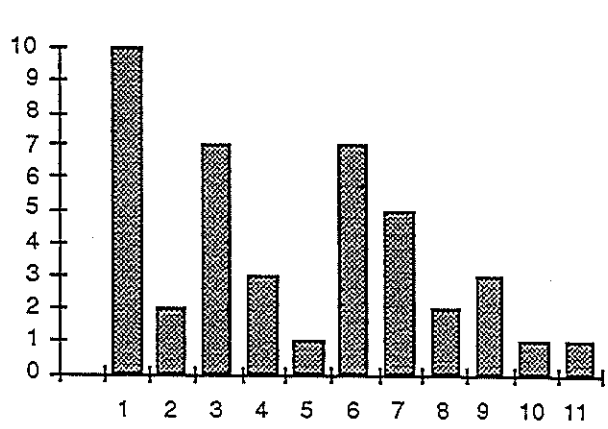


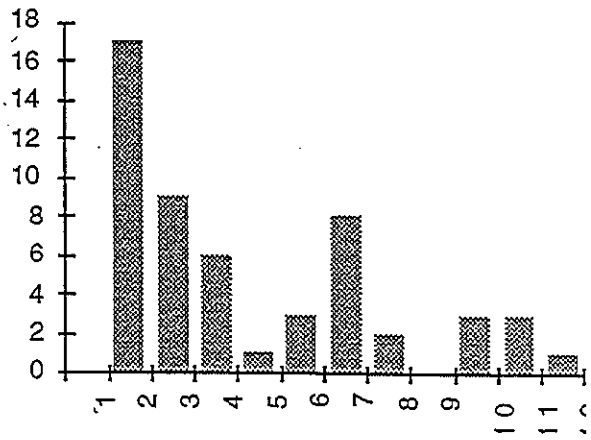


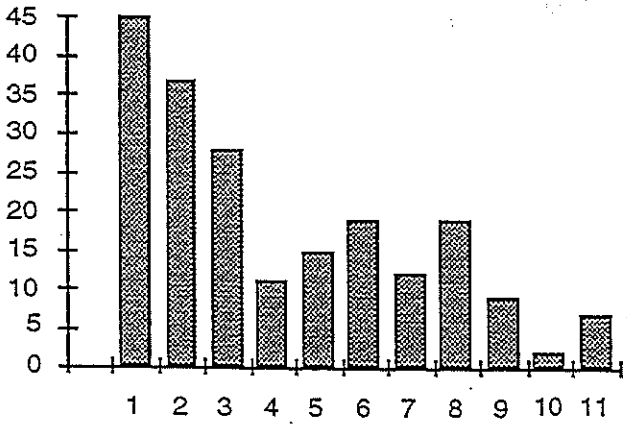


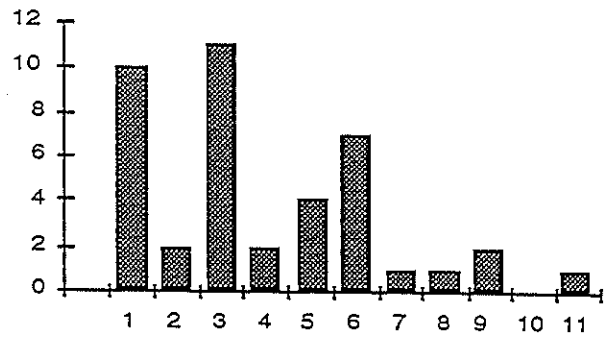
These are enharmonic histograms, that is, C# = Db.

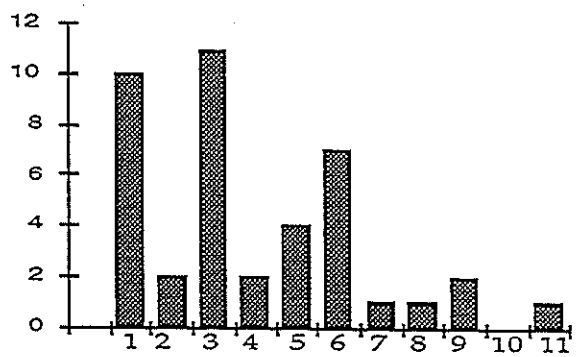
Interval histograms, non-tritone reduced, all sections





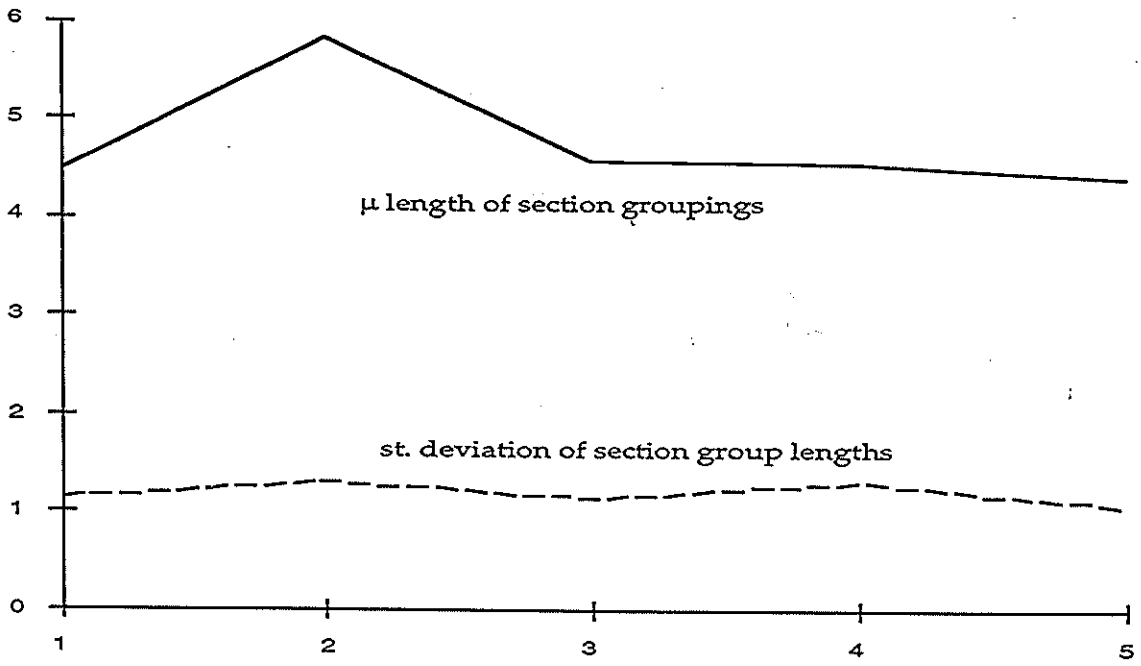






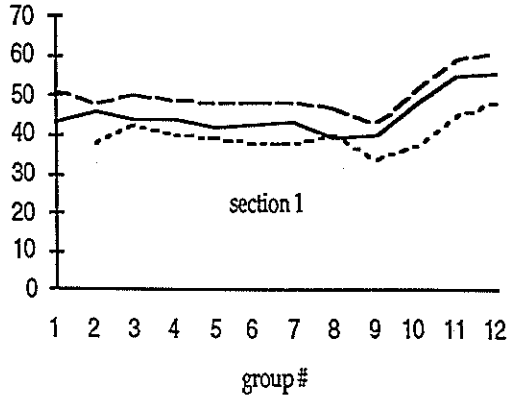
Treats inversions of intervals around the tritone as separate intervals, i.e maj 7th ≠ min 2nd.

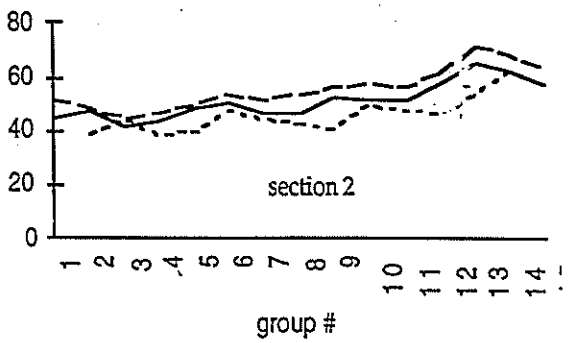
Lengths of different sections and groups
(in numbers of notes)

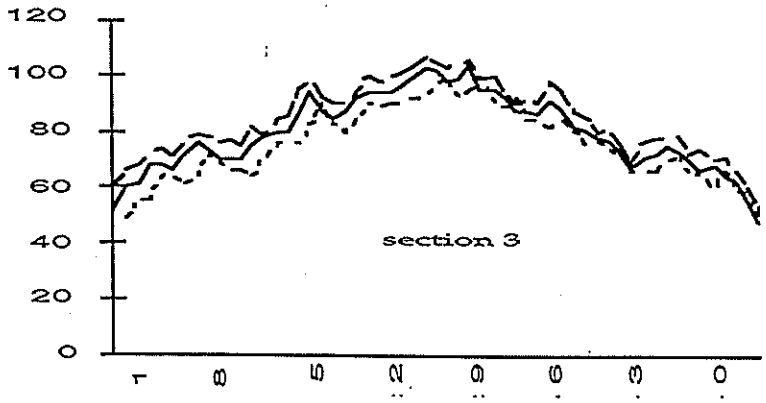


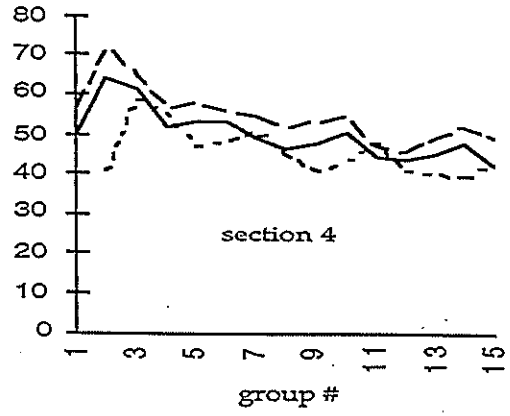
Section #	Total length	μ	δ
1	54	4.5	1.17
2	68	5.83	1.30
3	261	4.58	1.164
4	68	4.53	1.30
5	53	4.416	1.083

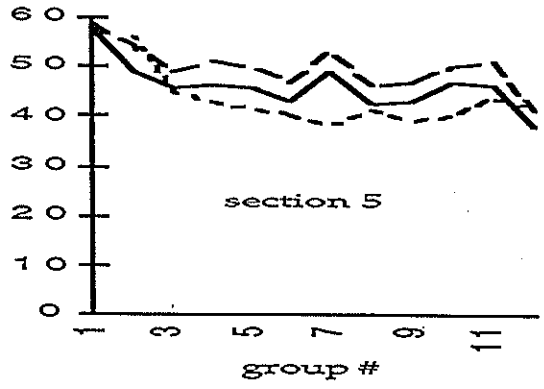
Mean and ranges of pitches for groups in 5 sections



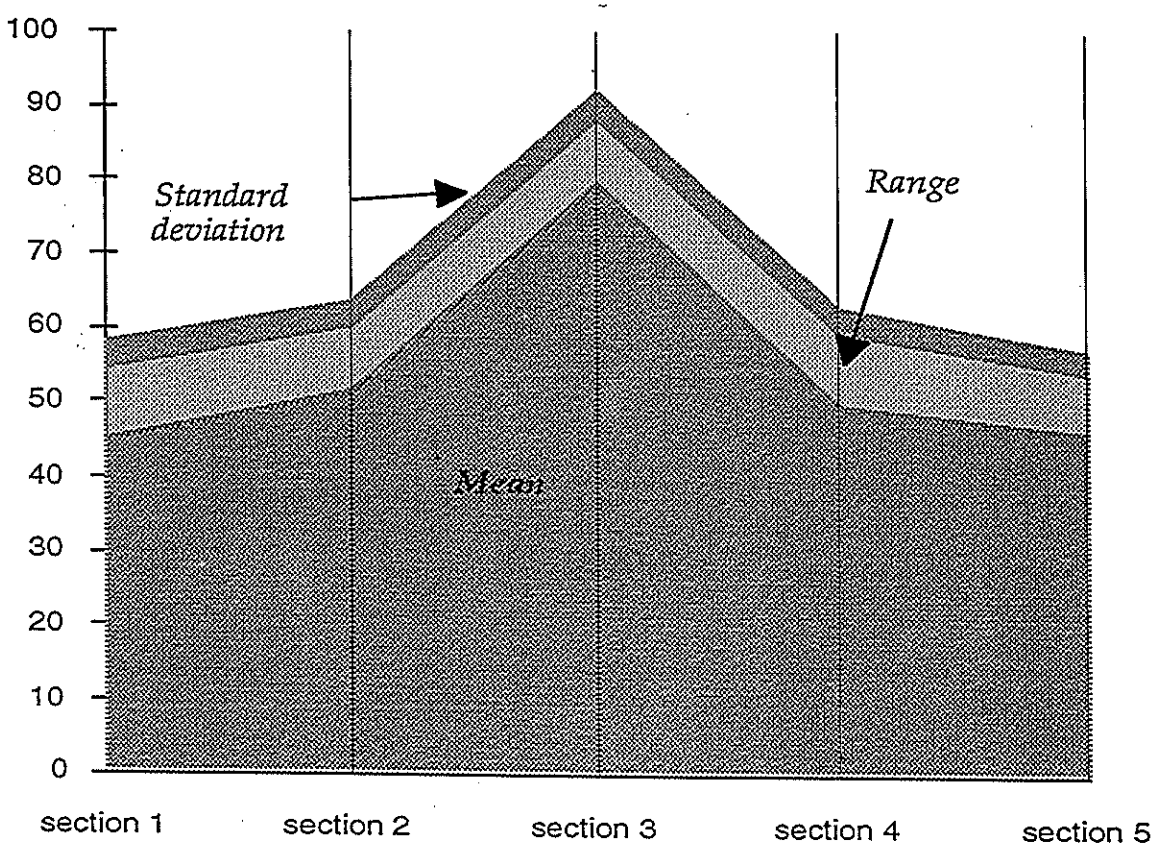




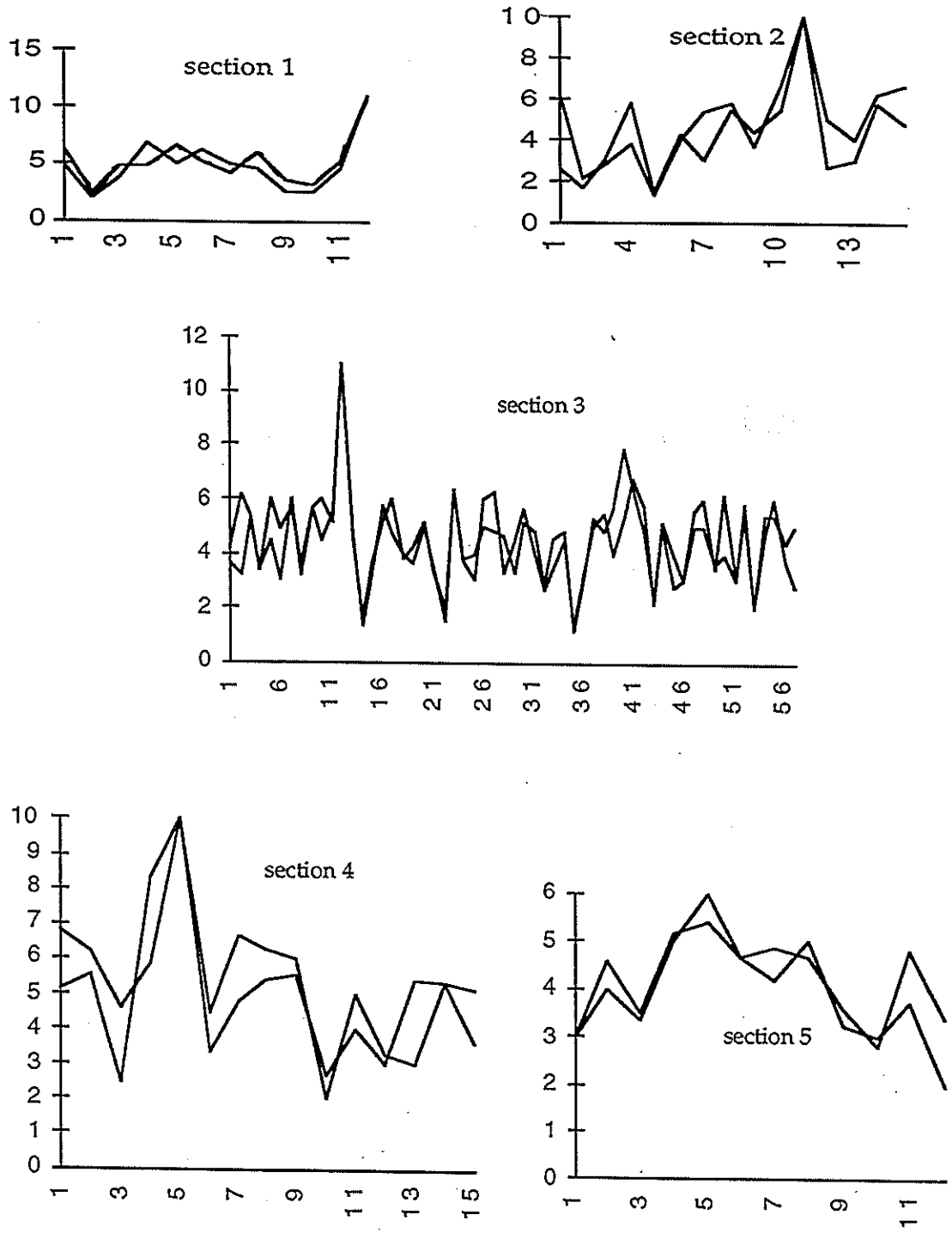




Pitch mean, range and standard deviation of sections, whole piece

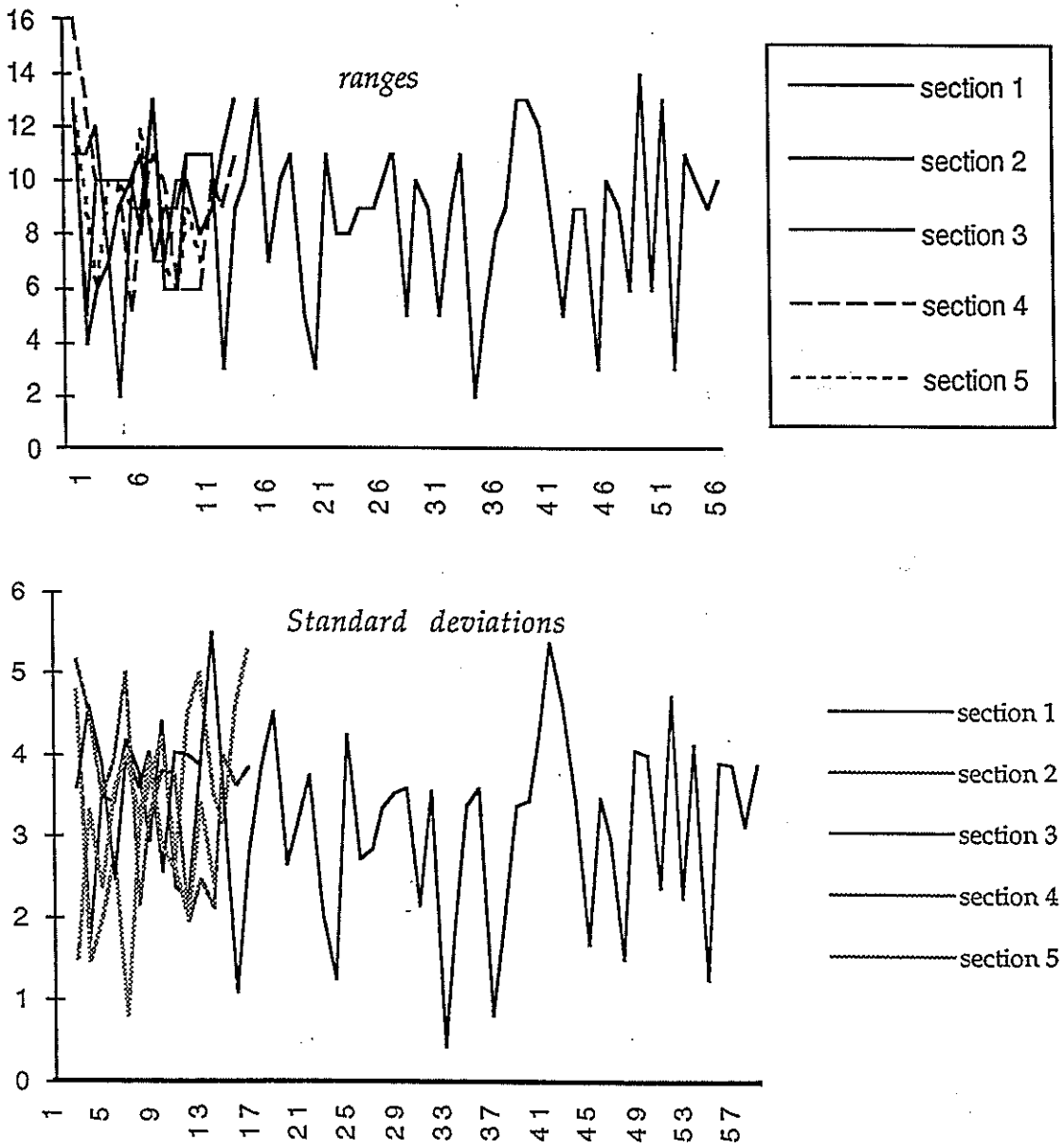


Linear and combinatorial interval means, each section



Linear means are the means of adjacent melodic intervals. Combinatorial means are the means of all intervals between all pitches for each group. Combinatorial means are of course, generally higher.

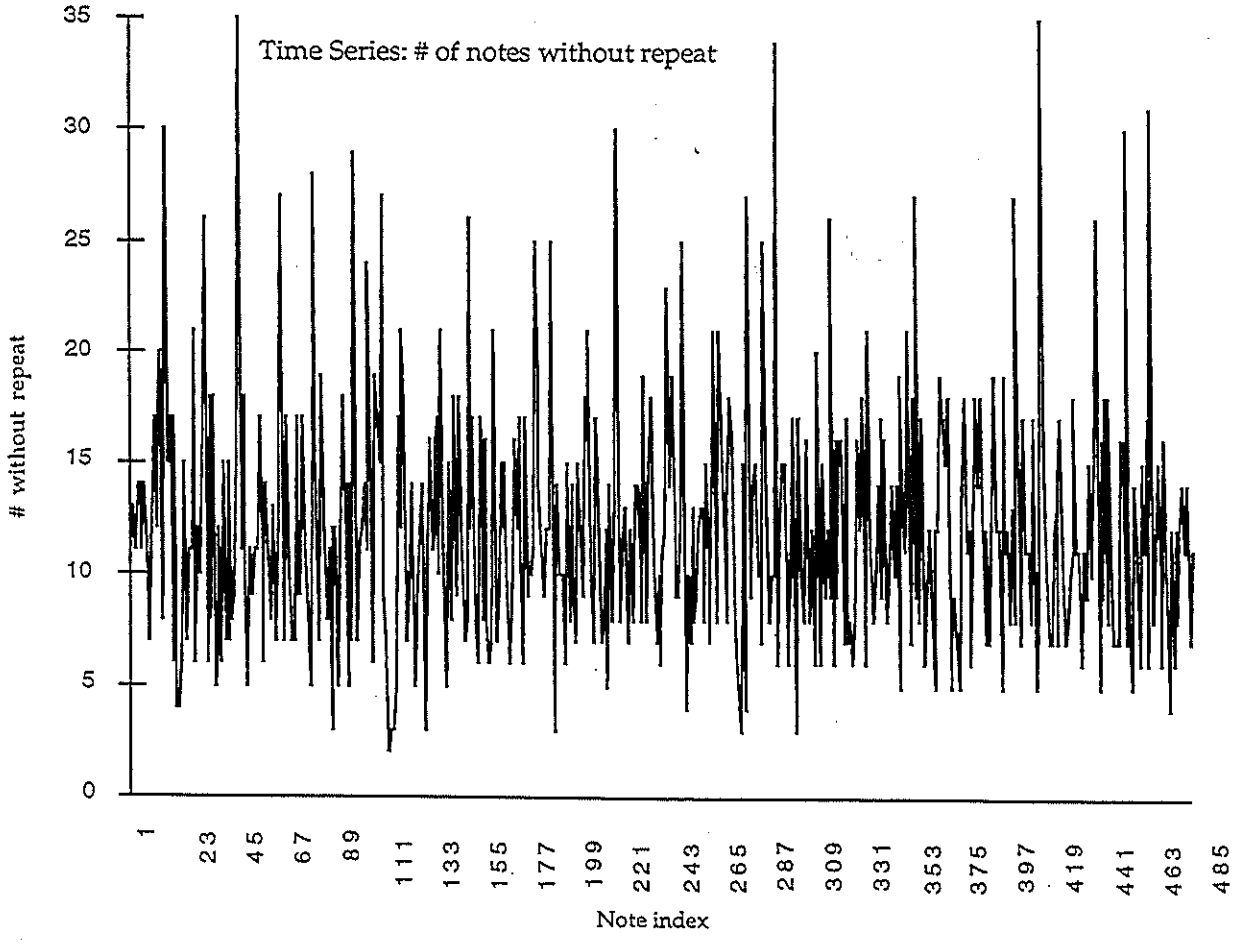
Pitch ranges and standard deviations of five sections, each group, superimposed



The shorter sections are superimposed over the longest, section 3, at the beginning of each time series.

Number of notes without repeating pitch class
(Time series representation: whole piece)

This time series illustrates the number of notes from a given note index before that pitch class is repeated.



Statistics for whole piece (number of notes from a given note without repeating a pitch class):

$\mu = 12.006$
$\delta = 5.548$

Sections start on note #'s:

0 54 122 383 451

Note periodicity of spikes, and large spikes near beginning and end of the work, as well as one more or less "near" the golden mean.

General equations for morphological metrics used

OLD (ordered linear direction) metric

$$\frac{\sum_{i=1}^{L-1} diff(sgn(\Delta(M_i,M_{i+1})), sgn(\Delta(N_i,N_{i+1})))}{L\text{-}1}$$

OCD (ordered combinatorial direction) metric

$$\frac{\sum_{j=1}^{L-1} \sum_{i=1}^{L-j} diff\ (sgn\ (\Delta(M_iM_j),- \Delta(N_iN_j))}{L_m}$$

UCD (unordered combinatorial direction) metric

$$\frac{\sum_{v=(-1,0,1)} |\#_vM - \#_vN|}{(L_m * 2)}$$

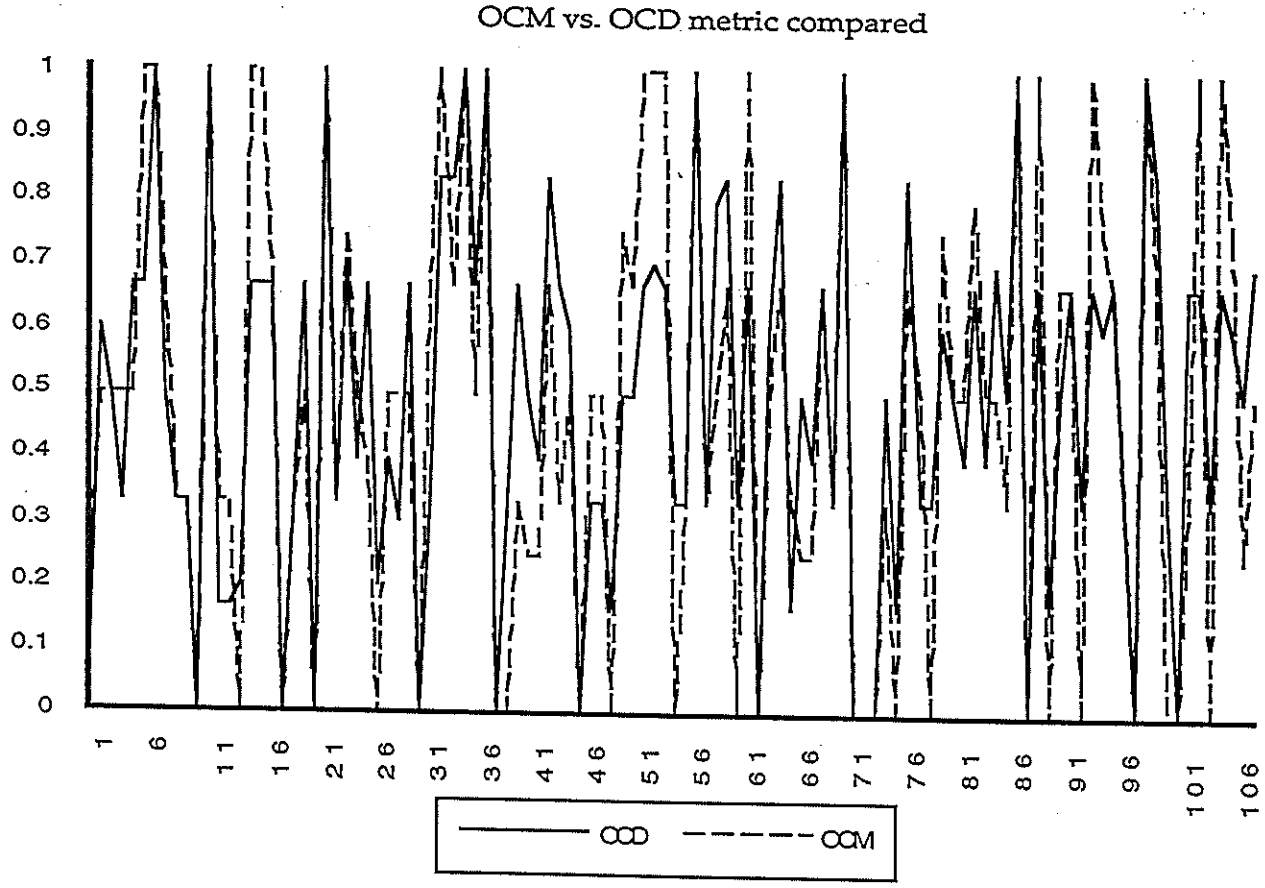
OCM (ordered combinatorial magnitude) metric

$$\frac{\sum_{j=1}^{L-1} \sum_{i=1}^{L-j} |\Delta(M_iM_j),- \Delta(N_iN_j)|}{L_m}$$

— L_m is the 2nd order binomial coefficient of the length of the morphs, or number of non-redundant pairwise relationships.
— Δ is some arbitrary interval function (like absolute value of pitch distance).
— *sign* is the standard up, down, equal contour function.
— *diff* is the statiscal difference between the "contour vectors" of the two morphologies, i.e the number of ups, downs, and equals (in the unordered metric), or the differences between corresponding cells (in the ordered one).

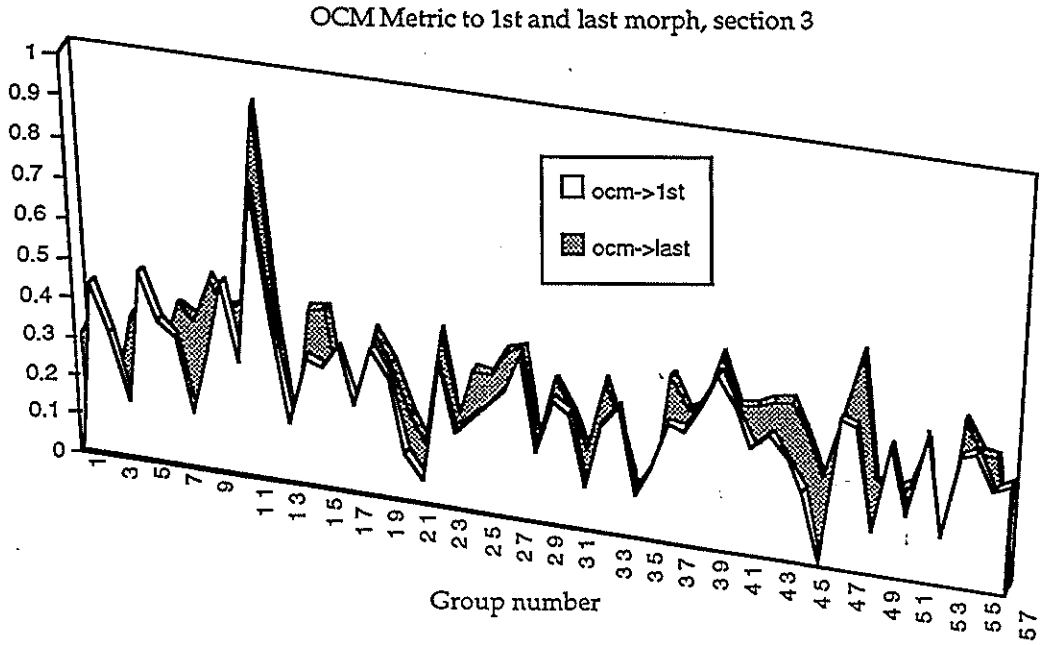
These are highly generalized versions of metrics, without scaling or weighting functions. These metric equations can take a great many forms, depending on the scaling and weighting functions used, ways that length differences are handled, and in types of interval function. In the examples in this talk, all length differences are truncated to the smaller morph (the simplest of about five possible algorithms for this).

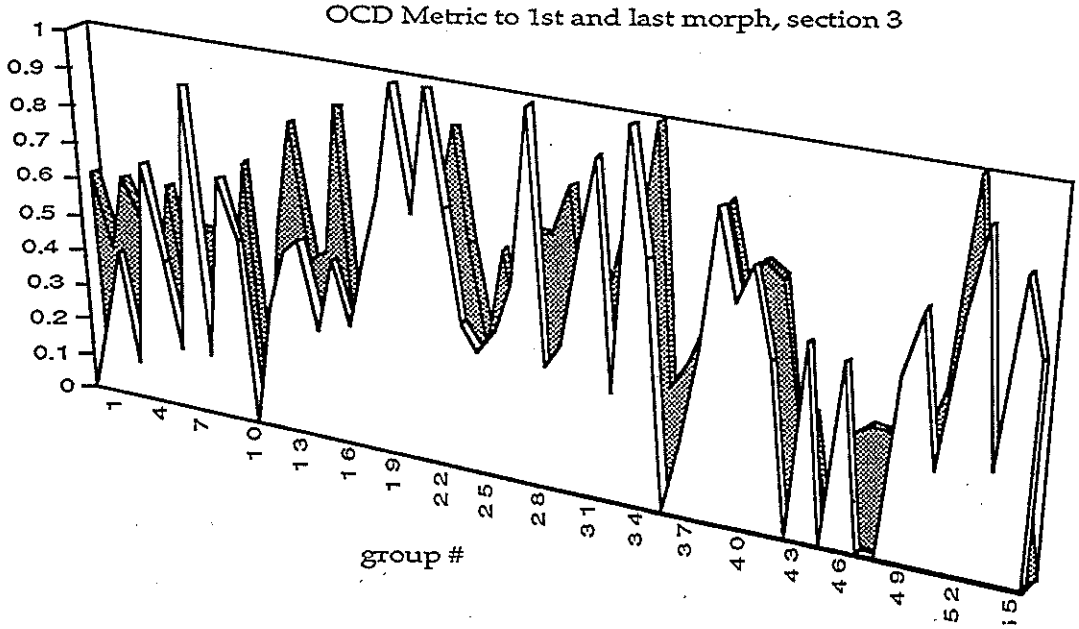
OCD and OCM metrics compared, adjacent groups, entire piece (directional vs. magnitude)



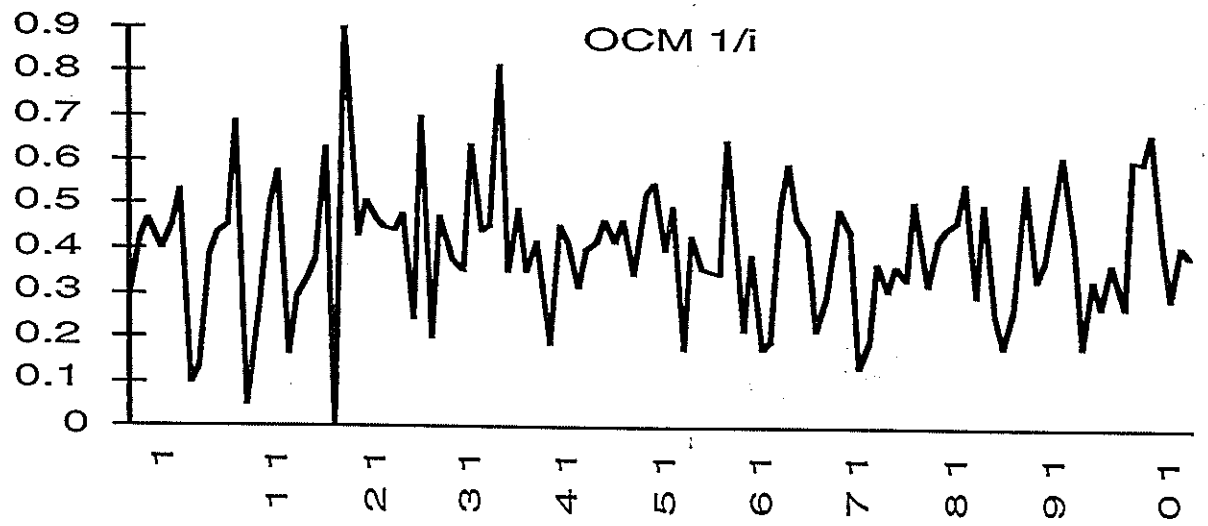
OCM much greater spread. OCD is more even between adjacent groups. That is, magnitude changes more than contour?

"2d" Metric comparisons, section 3

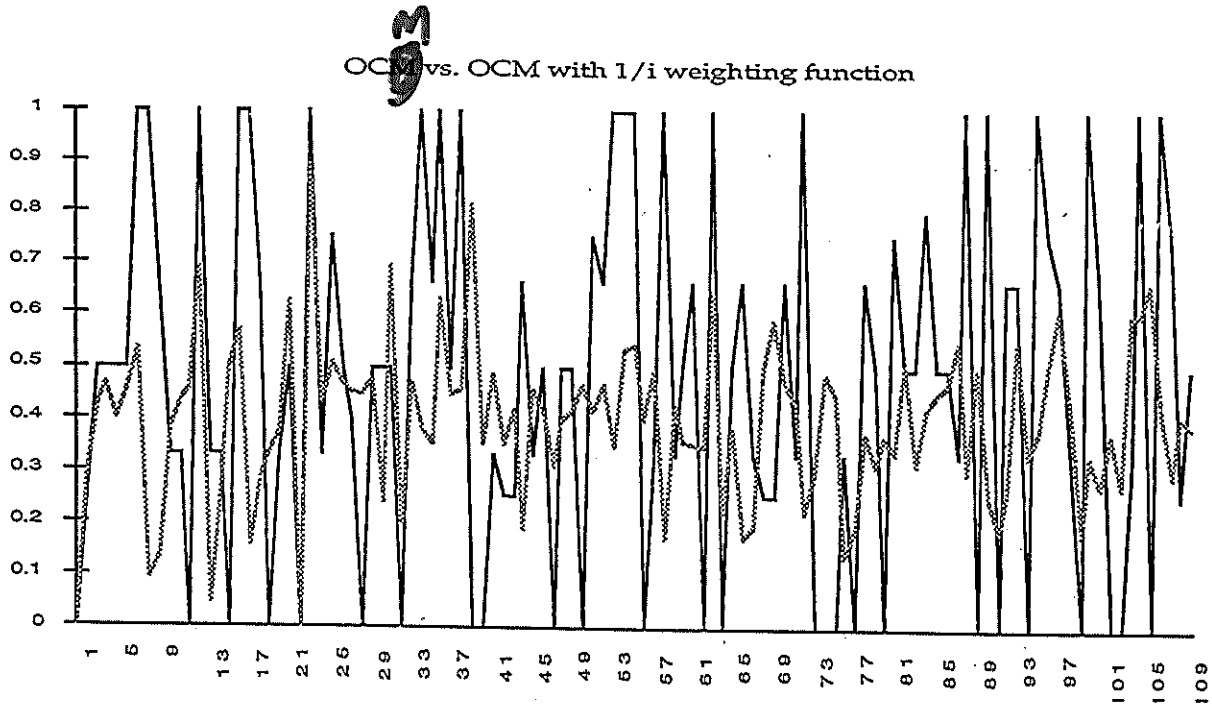




Ordered Combinatorial Magnitue (OCM) metrics compared, adjacent groups, entire piece (same metric different weighting functions)



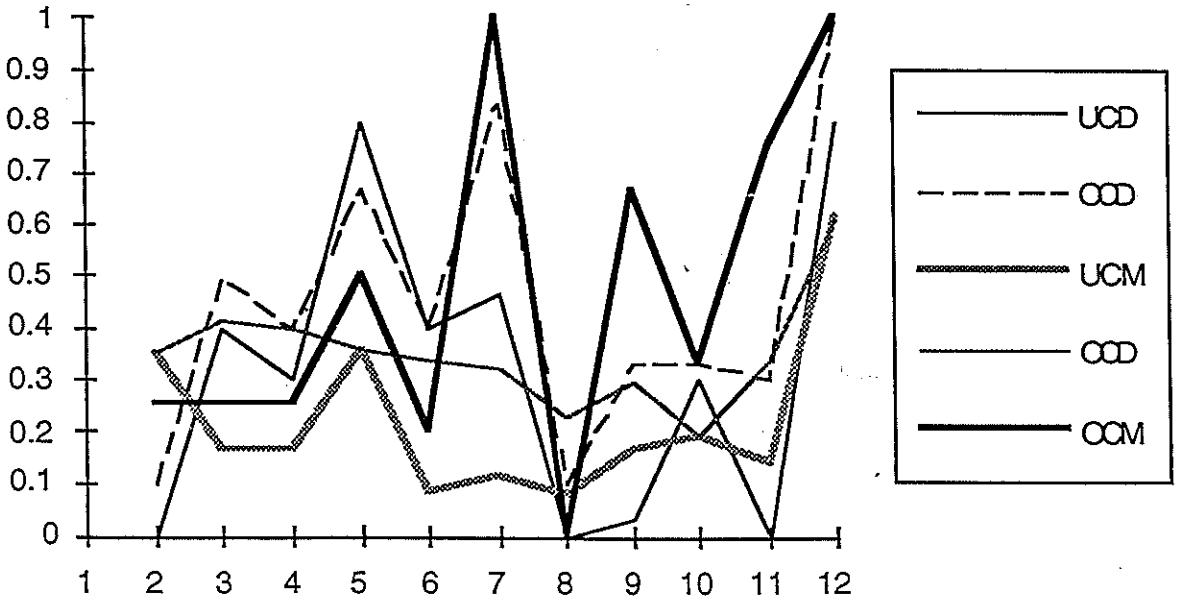
Superimposing OCM with 1/i weighting function on OCM with no weighting function:



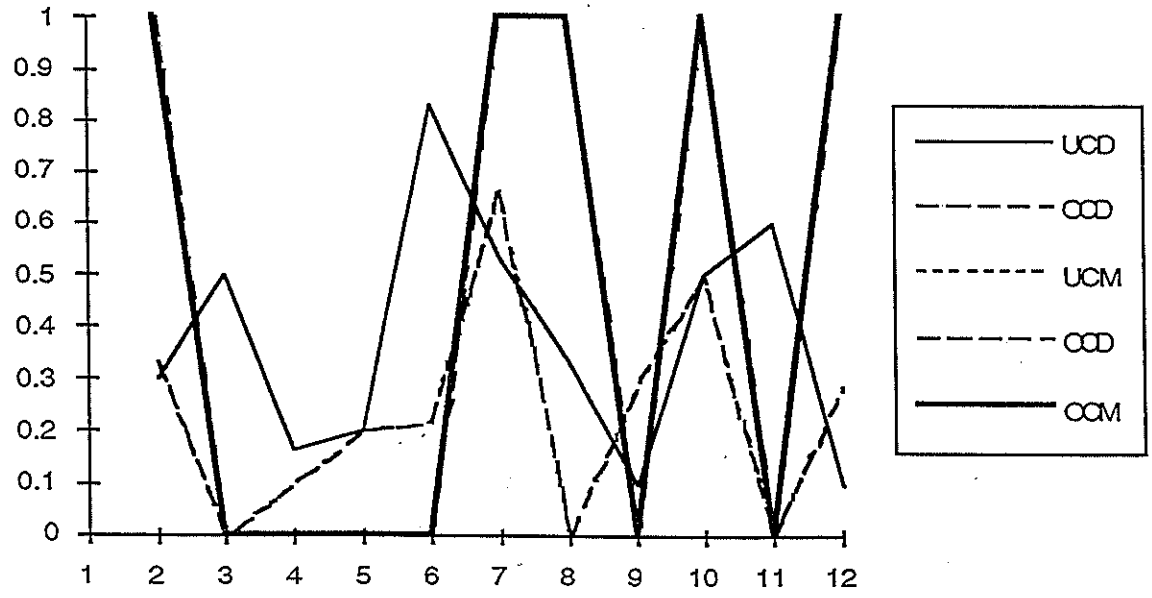
1/i weighting function linearly decreases the importance of intervals proportional to their order difference, as well as decreasing the importance of intervals to higher indices. In other words, the interval between the 1st and 3rd elements is weighted higher than that between the 1st and 4th, and all intervals to the 1st element are weighted higher than all intervals to the 2nd.

Five different metrics, compared on one section
(sections 1 and 5, "mirror images")

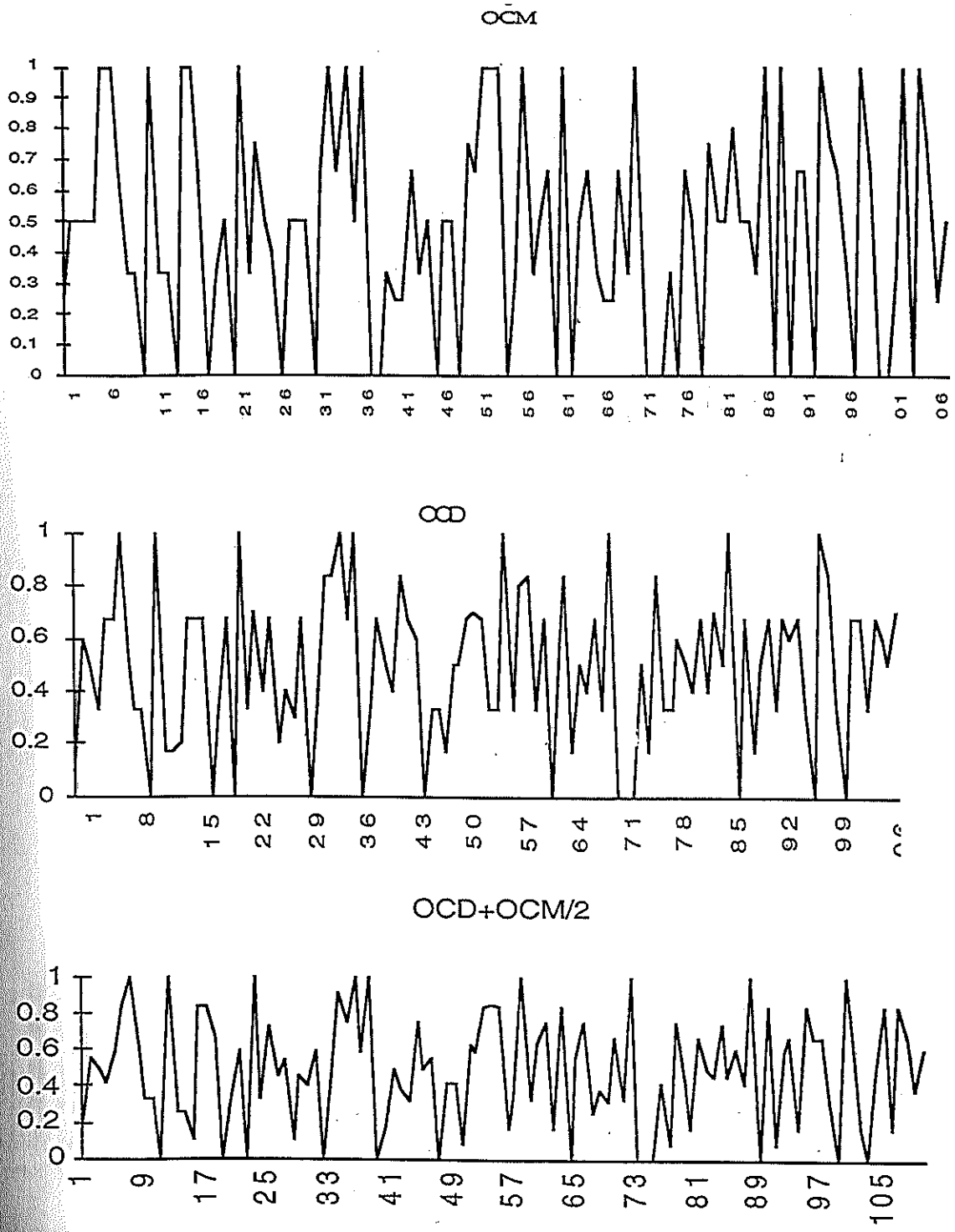
Five Metrics, Section 1, Adjacent groups



Five metrics, section 5, adjacent groups

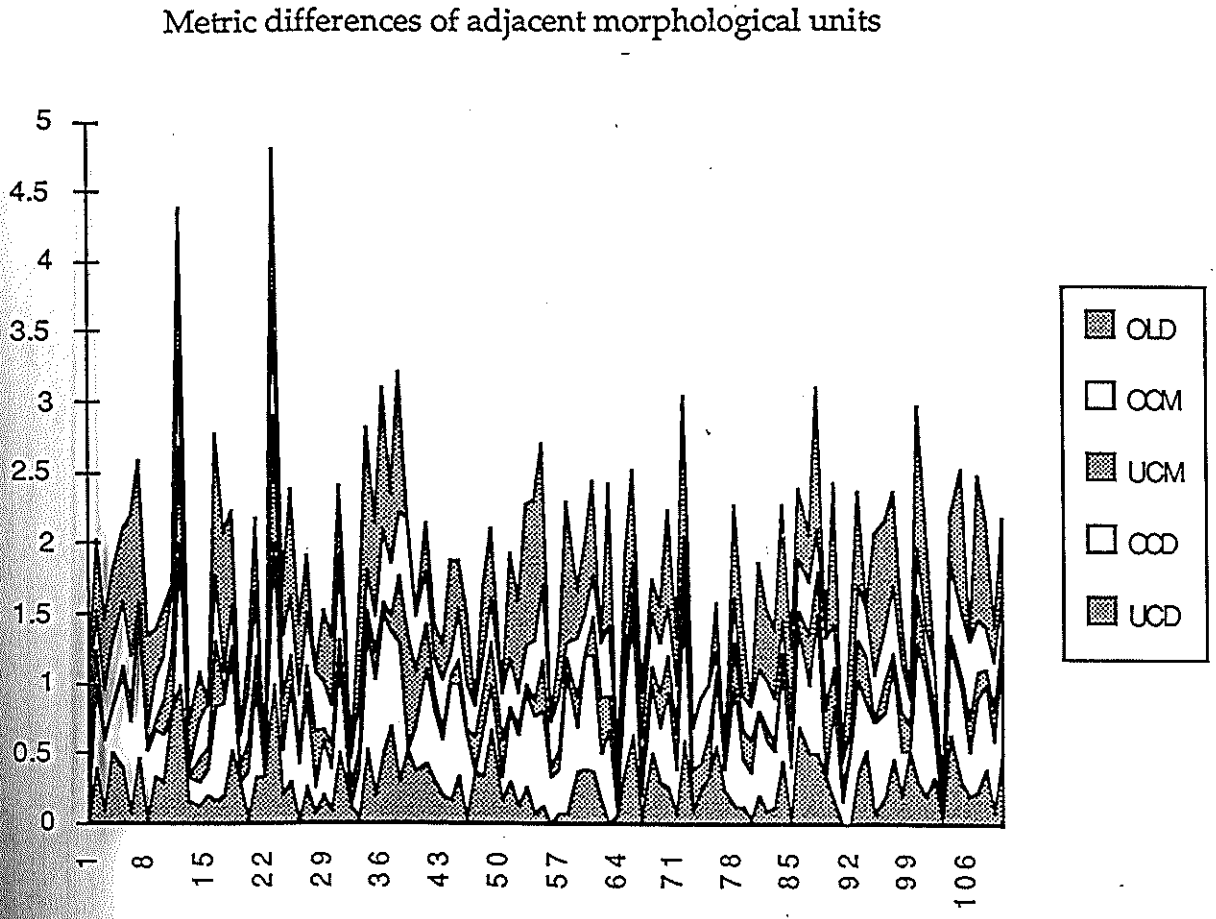


OCD, OCM Metrics, adjacent groups, entire piece
(Ordered combinatorial direction, ordered combinatorial magnitude)

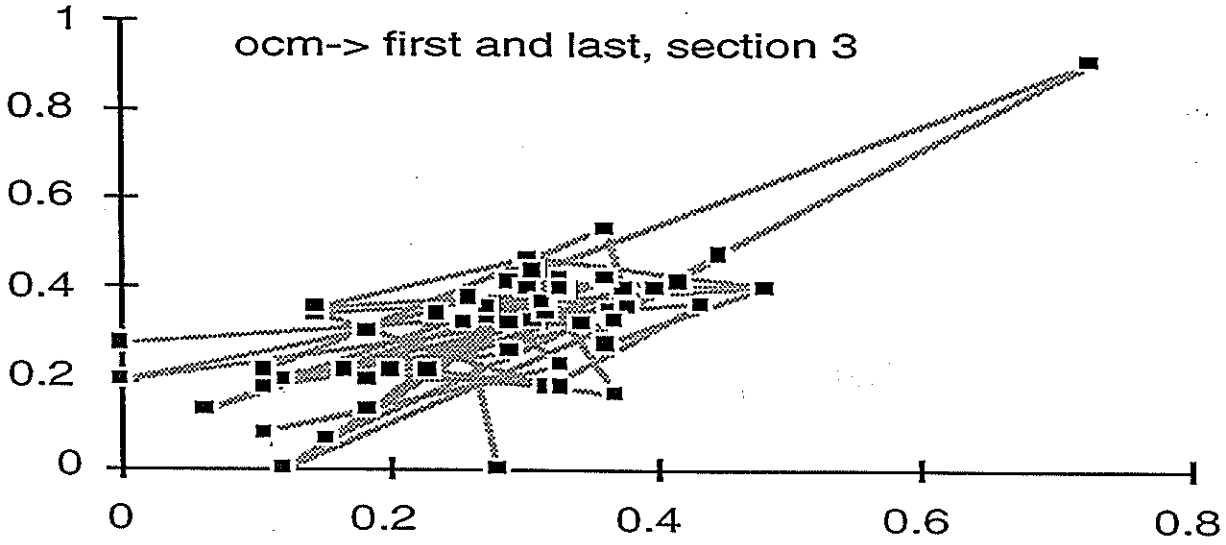


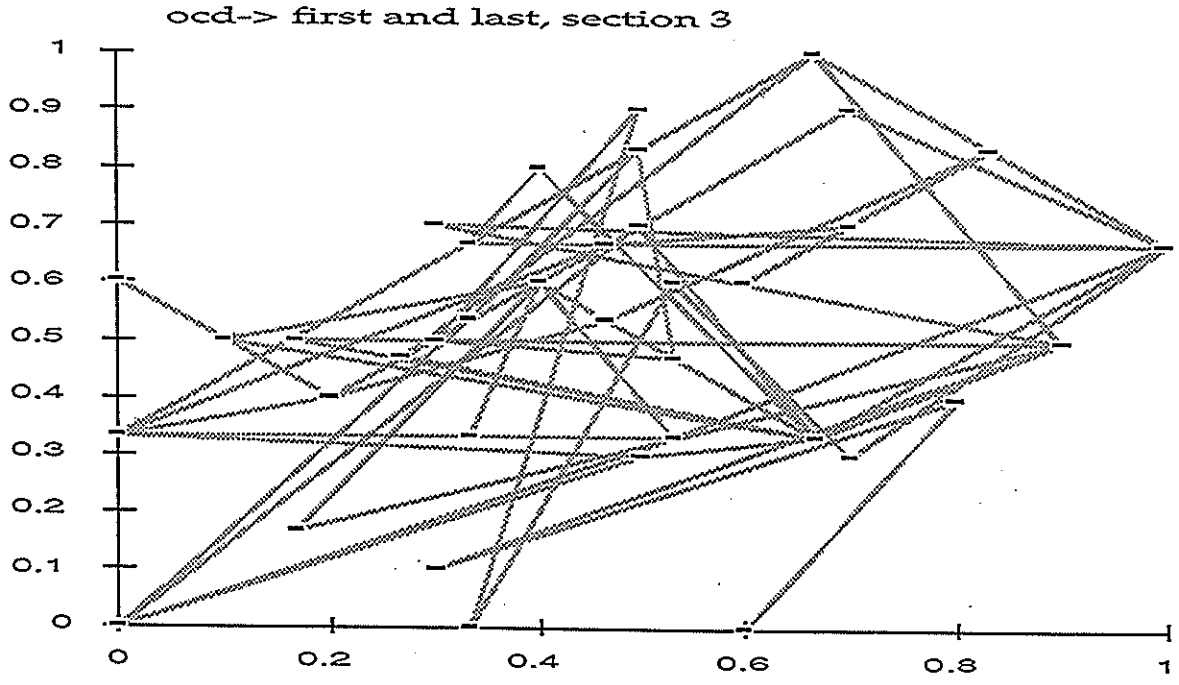
These metric functions measure similarity, by the given functions, on adjacent melodic groupings for the entire piece.

Five different morphological metrics, adjacent groups

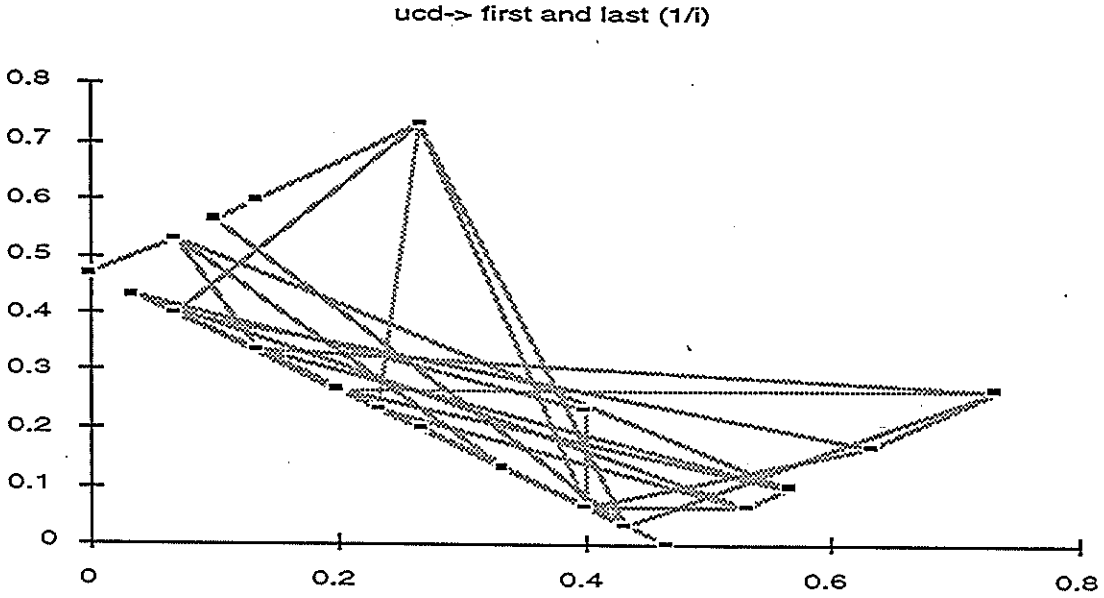


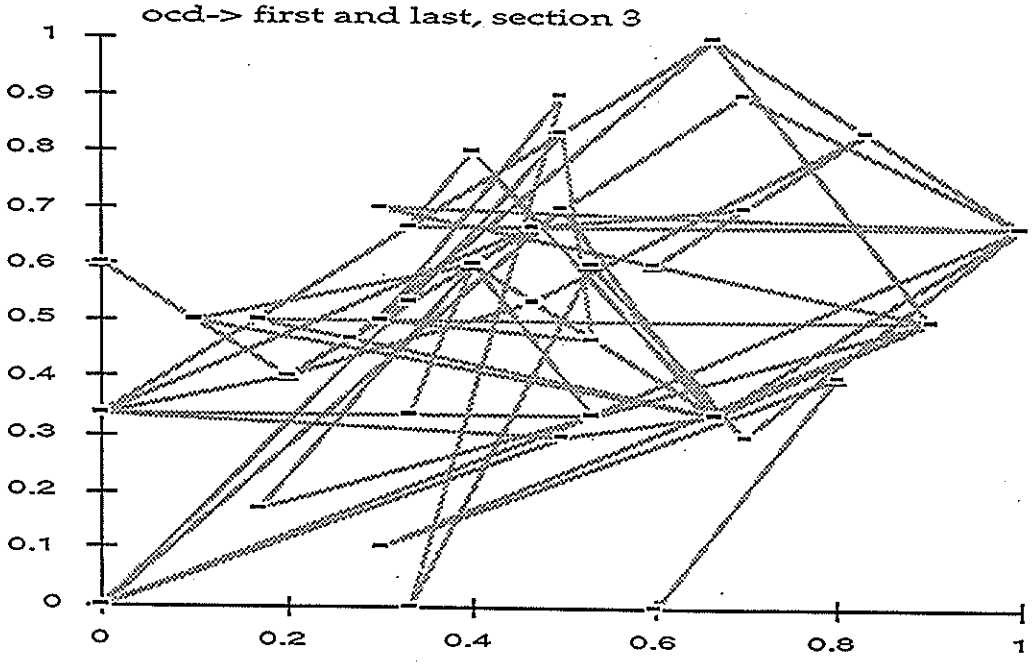
"2d" Scatter Plots, three different metric, section 3, continued
(1st metric uses a 1/i weighting function, section 2 use unit functions)





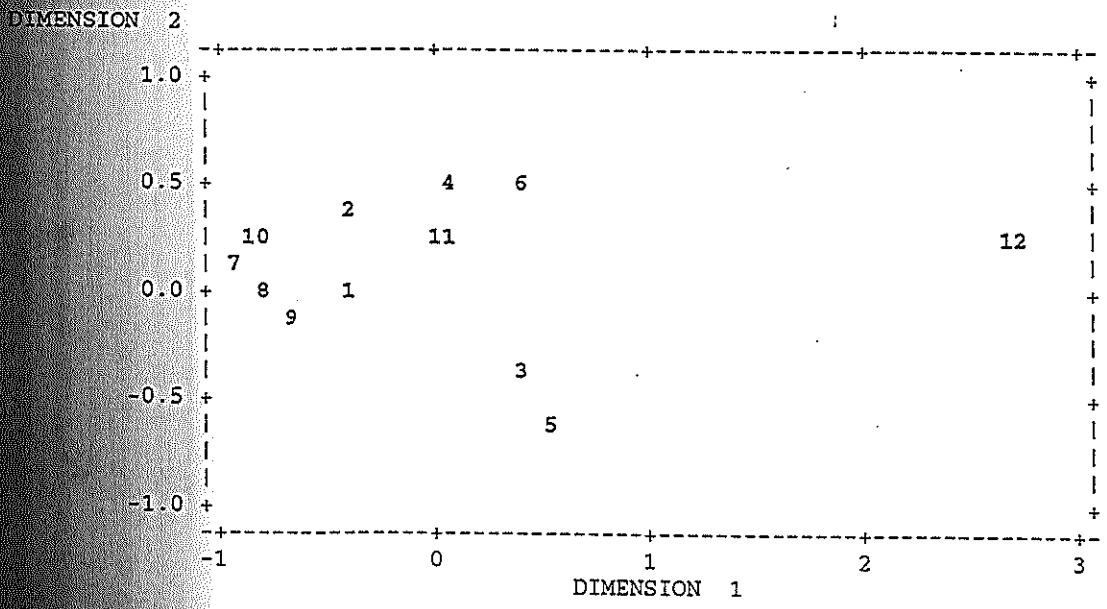
"2d" Scatter Plots, three different metric, section 3
(1st metric uses a 1/i weighting function, section 2 use unit functions)





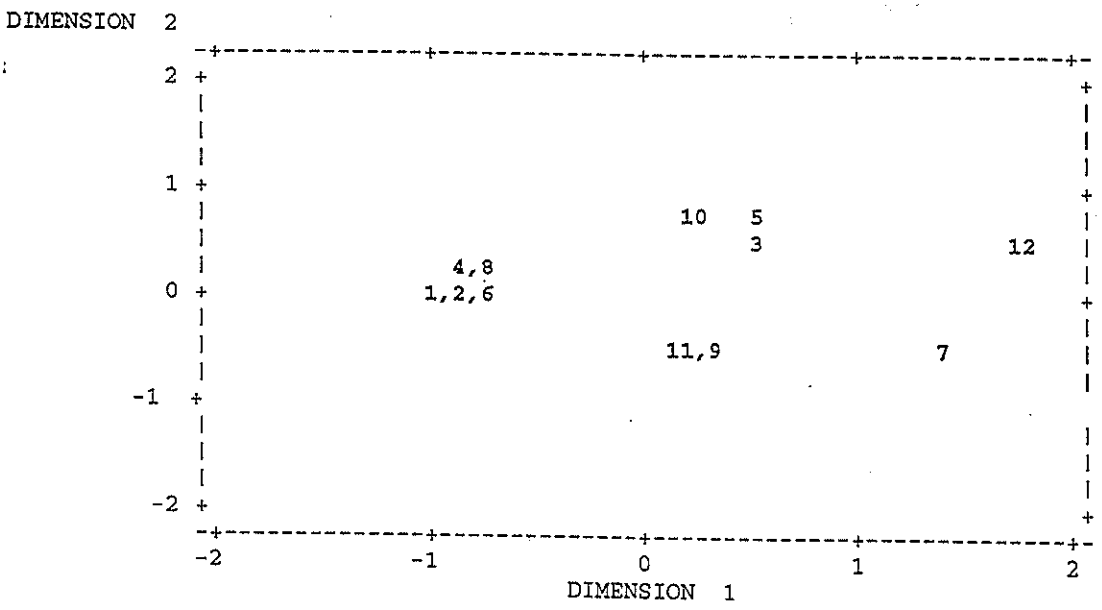
Multidimensional scaling coordinates in 2 Dimensions: OCM Metric (section 1)

Group	Plot	Dimension 1	Dimension 2
Group1	1	-0.42	-0.08
Group2	2	-0.40	0.25
Group3	3	0.37	-0.48
Group4	4	0.08	0.41
Group5	5	0.55	-0.70
Group6	6	0.39	0.39
Group7	7	-0.91	0.04
Group8	8	-0.83	-0.11
Group9	9	-0.65	-0.22
Group10	10	-0.85	0.14
Group11	11	0.01	0.22
Group12	12	2.67	0.14



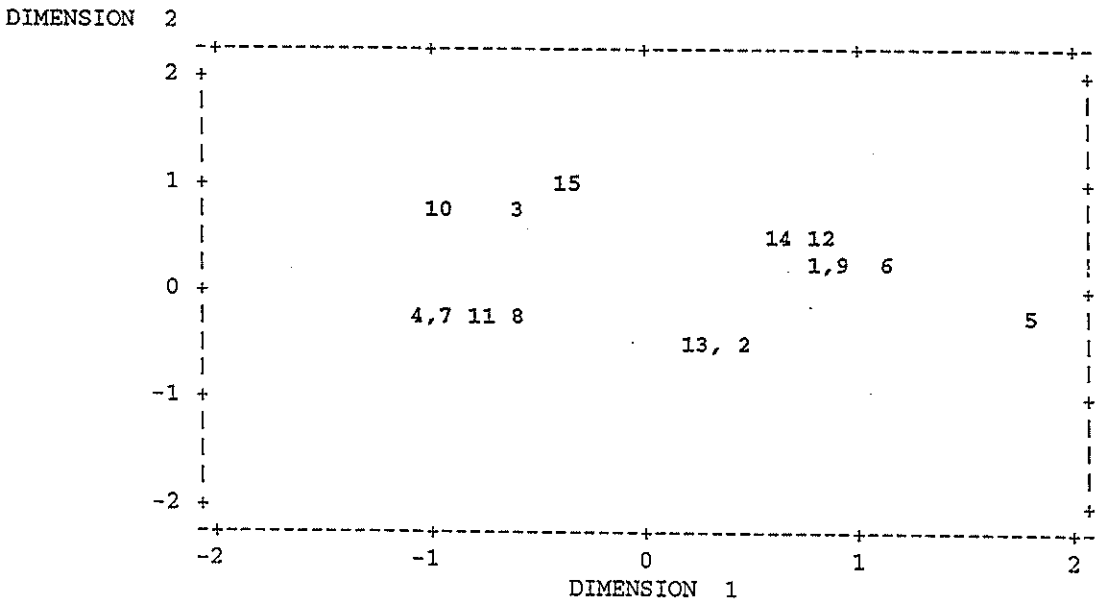
Multidimensional scaling coordinates in 2 Dimensions: OCD Metric (section 1)

Group	Plot	Dimension 1	Dimension 2
Group1	1	-0.94	-0.07
Group2	2	-0.97	-0.06
Group3	3	0.52	0.48
Group4	4	-0.88	0.12
Group5	5	0.45	0.68
Group6	6	-0.96	-0.22
Group7	7	1.33	-0.73
Group8	8	-0.83	0.21
Group9	9	0.19	-0.60
Group10	10	0.18	0.50
Group11	11	0.15	-0.59
Group12	12	1.75	0.27



Multidimensional scaling coordinates in 2 Dimensions: OCD Metric (section 2), unity scaling

Group	Plot	Dimension-	
		1	2
Group1	1	0.82	0.02
Group2	2	0.17	-0.61
Group3	3	-0.62	0.74
Group4	4	-1.05	-0.25
Group5	5	1.67	-0.31
Group6	6	1.00	0.08
Group7	7	-1.05	-0.25
Group8	8	-0.87	-0.46
Group9	9	0.78	0.18
Group10	10	-1.02	0.50
Group11	11	-1.03	-0.41
Group12	12	0.78	0.31
Group13	13	0.22	-0.75
Group14	14	0.61	0.41
Group15	15	-0.40	0.80



Multidimensional scaling coordinates in 2 Dimensions: OCD Metric (section 5), unity scaling

Group	Plot	Dimension 1	Dimension 2
Group1	1	1.80	-0.16
Group2	2	-1.13	-0.20
Group3	3	0.65	0.21
Group4	4	0.32	-0.43
Group5	5	0.33	-0.48
Group6	6	0.87	0.82
Group7	7	-0.91	0.63
Group8	8	-0.95	0.10
Group9	9	0.12	-0.79
Group10	10	-0.69	0.31
Group11	11	0.60	0.29
Group12	12	-1.00	-0.31