8 Schlesinger's harmoniai, Wilson's diaphonic cycles, and other similar constructs

THE HARMONIAI WERE proposed by the English musicologist Kathleen Schlesinger as a reconstruction and rediscovery of the original forms of the modal scales of classical Greek music. Schlesinger spent many years developing her theories by experimenting with facsimiles of ancient auloi found in archaeological sites in Egypt, Pompeii, and elsewhere. Later, she extended her studies to include flutes of ancient and modern folk cultures. As a result of her researches, she questioned the accepted interpretation of Greek musical notation. The results of these studies were previewed in a paper on Aristoxenus and Greek musical intervals (Schlesinger 1933) and were presented at length in her major work, *The Greek Aulos* (1939). Her writings are a major challenge to the traditional tetrachord-based doctrines of the Aristoxenian and Ptolemaic theorists. While there are compelling reasons to doubt that her scales were ever a part of Greek musical practice, they form a musical system of great ingenuity and potential utility in their own right.

This first part of this chapter is devoted to an exposition and analysis of her work. Various extensions and additions are proposed and near the end related materials, including Wilson's diaphonic cycles, are discussed.

The Schlesinger harmoniai

Schlesinger's harmoniai are 7-tone sections of the subharmonic series between members an octave apart. In theory, they are generated by aliquot divisions of the vibrating air columns of wind instruments. The same intervals, however, are obtained by the linear division of half strings. As string lengths are conceptually simpler than air columns, this discussion

8-1. *The diatonic Perfect Immutable System in the Dorian tonos according to Schlesinger. Each diatonic harmonia may be taken as an octave species of this system. (As elsewhere, at variance from Schlesinger, hypate meson is equated with E rather than F.) Trite synemmenon is required for the hypo-modes, in which it replaces paramese. The diatonic synemmenon tetrachord consists of the numbers 16 15 13 and 12.*

NOTE	M.D.	TRANS.
PROSLAMBANOMENOS	32	A
HYPATE HYPATON	28	B
PARHYPATE HYPATON	26	C
LICHANOS HYPATON	24	D
HYPATE MESON	22	E
PARHYPATE MESON	20	F
LICHANOS MESON	18	G
MESE	16	a
TRITE SYNEMMENON	15	b♭
PARAMESE	14	b
TRITE DIEZEUGMENON	13	c
PARANETE DIEZEUGMENON	12	d
NETE DIEZEUGMENON	11	e
TRITE HYPERBOLAION	10	f
PARANETE HYPERBOLAION	9	g
NETE HYPERBOLAION	8	a'

will refer to the former for clarity. The numbers or *modal determinants* assigned to each of the notes are to be understood as the denominators of ratios. The sequence 22 20 18 16 is a shorthand for the notes 22/22 22/20 22/18 22/16 or 1/1 11/10 11/9 11/8 above the tonic note 22.

The octave rather than the tetrachord is the fundamental module of these scales. Although the scales can be analyzed into tetrachords and disjunctive tones, the tetrachords are of different sizes which, in general, do not equal 4/3. Furthermore, each interval of the scale is different; the series of duplicated conjunct and disjunct tetrachords of the traditional theorists (chapter 6) is replaced by modal heptachords which repeat only at the octave.

The familiar names for the octave species are retained, but each modal octave is, in effect, another segment of the subharmonic series, bounded by a different modal determinant and its octave. 8-1 shows the form the Perfect Immutable System in the diatonic genus takes in her theory.

The modal determinants have many of the functions of tonics. As such, they serve to identify and define the harmoniai. Schlesinger also considers that mese itself has tonic functions, a point which is controversial even in the standard theory (Winnington-Ingram 1936).

The relations the other octave species have to the central Dorian octave is shown in 8-2. The seven harmoniai may also be constructed on a common tone, proslambanomenos, by assigning their modal determinants to hypate meson. In this case, there are six additional keys or tonoi which are named after the homonymous harmoniai. The Dorian and the other modal octaves are then found at corresponding transpositional levels in each tonos. Con-

8-2. *The diatonic harmoniai as octave species of the Perfect Immutable System in the Dorian tonos. Other tonoi are defined by assigning their modal determinants to hypate meson and proceeding through the subharmonic series. The Dorian, however, is the basis for Schlesinger's theory.*

	PS	HH	PH	LH	HM	PM	LM	M	TS	PM	TD	PD	ND	TH	PN	NH
	32	28	26	24	22	20	18	16	15	14	13	12	11	10	9	8
	A	B	C	D	E	F	G	a	b♭	b	c	d	e	f	g	a'
MIXOLYDIAN		28	26	24	22	20	18	16		14						
LYDIAN			26	24	22	20	18	16		14	13					
PHRYGIAN				24	22	20	18	16		14	13	12				
DORIAN					22	20	18	16		14	13	12	11			
HYPOLYDIAN						20	18	16	(15)	14	13	12	11	10		
HYPOPHRYGIAN							18	16	15		13	12	11	10	9	
HYPODORIAN								16	15		13	12	11	10	9	8

comitantly, there is a seven-fold differentiation of the tuning of the other notes of the Perfect Immutable System. These tonoi are shown in 8-3.

Anomalies and inconsistencies

The clarity and consistency of Schlesinger's system, however, is only apparent. Once one goes beyond the seven diatonic harmoniai, anomalies of various types soon appear.

Schlesinger explicitly denies harmonia status to the octave species running from proslambanomenos to mese, calling it the *bastard Hypodorian* or *Mixophrygian*. She rejects it because it resembles the Hypodorian an octave lower but differs in having 8/7 rather than 16/15 as its first interval. Yet this scale had a name (Hypermixolydian) in the standard theory and was rejected by Ptolemy precisely because it was merely the Hypodorian transposed by an octave.

Each of the diatonic harmoniai also had chromatic and enharmonic forms derived by subdividing the the first interval of each tetrachord and deleting the former mesopyknon. This process is identified with katapyknosis and is analogous to the derivation of the genera in the standard theory (see chapters 2 and 4). These forms are listed in 8-4 for the central octave of the Perfect Immutable System in each homonymous tonos.

It is also here that some of the most serious problems with her theory occur. Although all of the diatonic harmoniai occur as octave species of the Dorian, and of each other, the chromatic and enharmonic forms of the other harmoniai are not modes of the corresponding forms of the Dorian harmonia. Rather, they are derived by katapyknosis of the homonymous tonos. The symmetry is broken and the modes are no longer identical in

8-3. *Schlesinger's diatonic harmoniai as tonoi. Elsewhere she gives different forms, most notably variants of the Lydian, with 27 instread of 26, and Dorian, with 21 instead of 22 (Schlesinger 1939, 1–35, 142). A trite synemmenon could be defined in each tonos, but Schlesinger chose not to do so. Schlesinger conceived of the Hypolydian harmonia in two forms with 15 alternating with 14 (ibid., 26–27). Her theory demands that the Dorian trite synemmenon (15) be employed in all the hypo-modes, but she allows the alternation in the Hypolydian harmonia.*

	PS	HH	PH	LH	HM	PM	LM	M	PM	TD	PD	ND	TH	PH	NH
	A	B	C	D	E	F	G	a	b	c	d	e'	f'	g'	a'
MIXOLYDIAN	44	40	36	32	28	26	24	22	20	18	16	14	13	12	11
LYDIAN	40	36	32	28	26	24	22	20	18	16	14	13	12	11	10
PHRYGIAN	36	32	28	26	24	22	20	18	16	14	13	12	11	10	9
DORIAN	32	28	26	24	22	20	18	16	14	13	12	11	10	9	8
HYPOLYDIAN	28	26	24	22	20	18	16	15	13	12	11	10	9	8	7
HYPOPHRYGIAN	26	24	22	20	18	16	15	13	12	11	10	9	8	7	13/2
HYPODORIAN	24	22	20	18	16	15	13	12	11	10	9	8	7	13/2	6

different tonoi. Even the modal determinants of the harmoniai may be changed in different tonoi.

Other inconsistencies and anomalies may be noted. The chromatic and enharmonic forms are incompletely separated since the enharmonic and chromatic forms of some harmoniai share tetrachords. Even these presumed canonical forms do not agree with the varieties she derives elsewhere in *The Greek Aulos* from her interpretation of the Greek notation.

Because of certain irregularities in the notation, she claims that the modal determinant of the Lydian harmonia must have been altered at some period from 26 (13) to 27 and that of the Dorian from 22 to 21. These changes of modal determinants would not only have disrupted the tonal relations of the original harmoniai, but would also have affected the tonality of the rest of the system in all three genera. Since the Dorian harmonia was the center of the system, this would not have been a trivial change.

The question of modal determinant 15

Another problem is the status of 15 as a modal determinant. Schlesinger strongly denies the existence of a harmonia whose modal determinant is 15. Yet one of her facsimile instruments plays it easily. She also states that hypate hypaton could be tuned to 30 in the Hypodorian harmonia where it generates a perfectly good harmonia of modal determinant 15 with the octave at trite synemmenon (8-2).

The inclusion of modal determinant 15 is, on the whole, quite problematical. It enters originally as the Dorian trite synemmenon (B♭), the only accidental in the Greater Perfect System. Although Schlesinger mentions what she calls the conjunct Dorian harmonia where 15 substitutes for 14, and elsewhere allows 15 to freely alternate with 14, she uses trite syn-

8-4. *Schlesinger's chromatic and enharmonic harmoniai (Schlesinger 1939, 214). It is clear that these scales are not simply modes of the Dorian chromatic and enharmonic genera, but are derived from the homonymous tonoi. The chromatic and enharmonic forms are derived by two successive doublings of the modal determinant followed by note selection to obtain the desired melodic contours. The upper tetrachords of the chromatic and enharmonic forms of the Dorian and Hypolydian harmoniai are identical. In the Hypolydian harmonia 30 (15) may replace 28 (14). The Hypophrygian and Hypodorian harmoniai have a single enharmonic-chromatic form.*

HARMONIA	CHROMATIC	ENHARMONIC
MIXOLYDIAN	28 27 26 22 20 19 18 14	56 55 54 44 40 39 38 28
LYDIAN	26 25 24 20 18 17 16 13	52 51 50 40 36 35 34 26
PHRYGIAN	24 23 22 18 16 15 14 12	48 47 46 36 32 31 30 24
DORIAN	44 42 40 32 28 27 26 22	44 43 42 32 28 27 26 22
HYPOLYDIAN	40 38 36 28 26 25 24 20	40 39 38 28 26 25 24 20
HYPOPHRYGIAN	36 35 34 26 24 23 22 18	36 35 34 26 24 23 22 18
HYPODORIAN	32 31 30 24 22 21 20 16	32 31 30 24 22 21 20 16

emmenon mainly to construct the diatonic hypo-modes. This is very much at variance with the usage of this note by the standard theorists whose Hypodorian, Hypophrygian, and Hypolydian modes employ only the natural notes of Greater Perfect System.

For these theorists, trite synemmenon and the rest of the synemmenon tetrachord are part of the Lesser Perfect System and are used to primarily illustrate the melodic effect of modulations to the key a perfect fourth lower. Bacchios also employs it to illustrate certain rare intervals such as the ekbole, spondeiasmos, and eklysis (chapters 6 and 7). The combination of the Greater and Lesser Perfect Systems to form the Perfect Immutable System is basically a pedagogical device, not a reflection of musical practice. Furthermore, the Lesser Perfect System terminates with the synemmenon tetrachord, but to complete Schlesinger's hypo-harmoniai the note sequence would have to switch back into the notes of the Greater Perfect System. Although chromaticism and modulation occur both in theory and in the surviving fragments (Winnington-Ingram 1936), this use of synemmenon would seem to be most unusual.

Historical evidence

Much of Schlesinger's case for the harmoniai is based on fragmentary quotations from classical Greek writers. This evidence is dubious support at best.

Theorists such as Aristoxenos complain about the unstable pitch and indeterminate tuning of the aulos (Schlesinger 1939). Aristoxenos claims that the intervals of music are determined by the performance skill of the player on both stringed and blown instruments and not by the instruments themselves. This polemic may be interpreted either as referring to the inherent pitch instability of the instrument or to the difficulty of bending the pitches so as to approximate a scale system for which it is not physically suited, i.e. the standard tetrachordal theory. Whatever the correct interpretation, the passage does suggest that Schlesinger's harmoniai played little or no role in Greek musical practice in the fourth century BCE.

The problem lies with our ignorance of the Greek music and its mode of performance. It is quite possible for an instrument to be musically prominent and at the same time difficult to play in acceptable tune. Schlesinger may well have been right about the natural scales of auloi and still be entirely wrong about their employment in Greek music of any period.

The harmoniai in world music

Schlesinger also tries to bolster her argument by appealing to ethnomusicology. Her case for the employment of the harmoniai in non-European folk and art music gives the impression of overpleading, especially in her analysis of Indonesian tunings. It is true, however, that wind instruments from many cultures often have roughly equidistant, equal sized finger holes. For example, the scales of many Andean flutes do appear to resemble sequences of tones from the various harmoniai, although the scales may not be identical throughout the gamut (Ervin Wilson, personal communication). The scales on these instruments are usually pentatonic, rather than heptatonic. Often one or more tones will diverge from the heptatonic pattern, particularly with respect to the vent, which is tuned to bring out the pentatonic structure. Nevertheless, some of the harmoniai sound very similar to the scales heard on recordings of Bolivian and Peruvian music. Hence, these data may serve as at least a partial vindication of her ideas.

Empirical studies on instruments

In *The Greek Aulos*, Schlesinger made use of a large body of data obtained by constructing and playing facsimiles of ancient auloi. She also studied fipple flutes and other folk wind instruments. These studies deserve critical attention.

The chief difficulty one has in evaluating this work is its lack of replication by other investigators. However, there are two published experimental studies which are relevant to her hypotheses.

The first is that of Letter, who made the assumption that two of the holes on the surviving auloi were 4/3 or 2/1 apart (Letter 1969). From measurements on these instruments, he determined the probable reed lengths. His measurements and calculations yielded a number of known tetrachords, including 12/11 · 11/10 · 10/9, 9/8 · 88/81 · 12/11, 9/8 · 16/15 · 10/9, 14/13 · 8/7 · 13/12, and some pentachordal sequences, but little convincing evidence for the subharmonic series or the harmoniai.

More recently, Amos built modal flutes with holes spaced at increments of one-eighth the distance from the fipple to the open end and the studied the resulting intervals (Amos 1981). This procedure, however, is not really in accord with Schlesinger's work. She employed rather complex formulae involving corrections for the diameter and certain other physical parameters to determine the spacing of the holes of modal flutes.

The pitches of Amos's flutes were measured by audibly comparing the flute tone to a calibrated digital oscillator and minimizing beats. Amos's results show that the resulting intervals are subject to wide variation from flute to flute and depend upon humidity, wind pressure, fingering, and other parameters.

While not strictly comparable to Schlesinger's results, the results of these investigators suggest that one should be cautious in extrapolating the tuning of musical systems from the holes of wind instruments.

Schlesinger herself made the same caveat and stated that the aulos alone gave birth to the harmoniai. She claimed that the acoustical properties of the aulos are simpler than those of the flute, and therefore, one can accurately deduce the musical system from the spacing of the finger holes of auloi. People who have made and played aulos-like instruments are less certain.

Lou Harrison found the traditional Korean oboe, the piri (and the homemade miguk piri), to be difficult to play in tune and noted its tendency to overblow at the twelfth (personal communication). Jim French, who has spent a number of years researching the aulos from both an archaeological and an experimental perspective, has discovered that the type of reed and its processing are far more crucial than Schlesinger implies. His results with double auloi indicate that the selection of a particular reed can change the fundamental by a 4/3 (personal communication). Duplicated tetrachords are thus quite natural on this kind of instrument. He has also found that sequences of consecutive intervals from harmoniai such as that on 16 (Hypodorian) are relatively easy to play on these instruments and may be embodied in historical examples and artistic depictions.

Composition with the harmoniai

The question of whether or not Schlesinger's harmoniai are relevant to Greek or world music may be of less importance to the experimental musician than their possible use in composition. Her most fruitful contribution ultimately may be her suggestion that the harmonia be considered a "new language of music" (Schlesinger 1939).

Schlesinger tuned her piano to the Dorian harmonia in which C (at 256 Hertz) equals the modal determinant 22. Thus she used only an 11–pitch gamut. For some unstated reason, she did not give a tuning for the note B♭, which would have had the modal determinant 25, though she did include such prime numbers as 17 and 19 and composites of comparable size such as 22 and 24. One would think that the Phrygian harmonia on 24 would make more efficient use of the keyboard, unless there are problems with the altered tension of the piano strings. This, of course, would not be a limitation with electronic instruments.

Schlesinger was fortunately able to enlist the composer Elsie Hamilton from South Australia in these efforts. Hamilton composed a number of works in the Dorian diatonic tuning between 1916 and 1929. In 1935, Hamilton trained a chamber orchestra in Stuttgart to perform in the harmoniai. Although several orchestral and dramatic works were composed and performed during this period, it has been impossible to find further information about the composer or discover whether the scores are still extant.

From the excerpts in *The Greek Aulos*, it would appear that Hamilton employed a conservative melodic idiom with straightforward rhythms (8-6). Schlesinger comments that such a simplification was necessary for both "executant and listener." The quotations from the score of *Agave*, brief as they are, seem quite convincing musically in a realization on a retunable synthesizer.

Hamilton's harmonic system is of considerable interest. Although familiar chords are scarce in this system, virtually any interval larger than a melodic second is at least a quasi-consonance. Rather than attempt a translation of tertian harmonic concepts to this tuning, Hamilton instead chose to use the tetrachordal frameworks of the modes as the basic consonances (8-5 and 8-6a). In the Dorian mode, this chord would be 22 16 14 11 (1/1 11/8 11/7 2/1), with 15 (22/15) as an alternative tone.

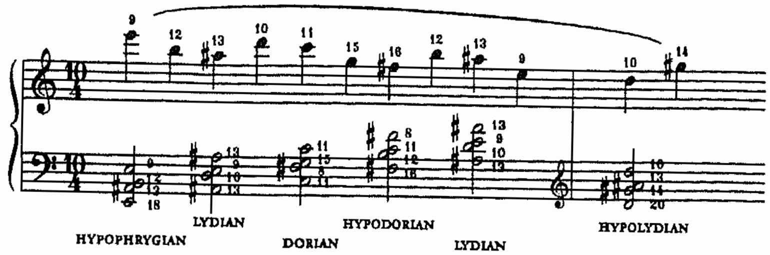
A melodic line may be supported by a succession of such chords taken from all seven of the modes. Hamilton augmented this somewhat sparse

8-5. *Harmonization of Schlesinger's harmoniai. Tetrachordal framework chords. Chords from the "conjunct" harmoniai in which 15 replaces 14 are also shown where applicable.*

	DISJUNCT	CONJUNCT
MIXOLYDIAN	28:22:20:14	28:22:16:14
LYDIAN	26:20:18:13	26:20:14:13, 26:20:15:13
PHRYGIAN	24:18:16:12	24:18:13:12
DORIAN	22:16:14:11, 22:16:15:11	22:16:12:11
HYPOLYDIAN	20:15:13:10, 20:14:13:10	20:15:11:10, 20:14:11:10
HYPOPHRYGIAN	18:13:12:9	18:13:10:9
HYPODORIAN	16:12:11:8	16:12:9:8

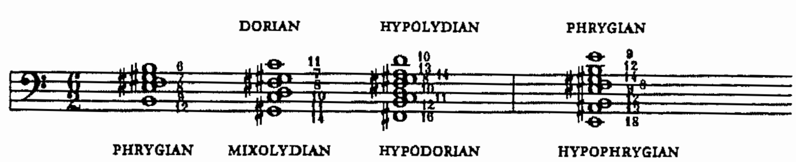
8-6. *Excerpts from* Agave *by Elsie Hamilton, with ratio numbers.*

(a) Tetrachordal framework chords ("Sunrise").

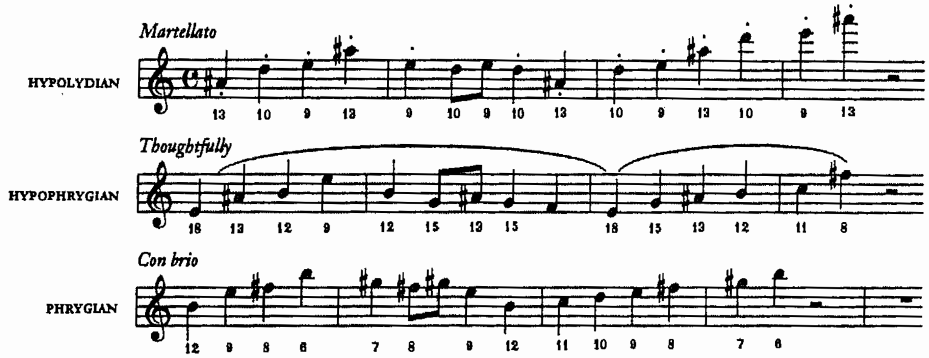


(b) Mixed chorus and tetrachords of resolution ("Funeral March").

(c) Combined framework chords ("Sunrise").



(d) Modal tranposition.



vocabulary with chords formed by the union and intersection of chords from two related harmoniai (8-6b, 8-6c, and 8-7). In the latter case, the chords are resolved to their common dyad.

She also discovered that parallel transposition results in changes of modality which are musically exploitable (8-6d), although the given examples are stated to have been approximated to the piano intonation.

One would characterize her harmonic techniques as essentially polytonal and polymodal, rather than "diatonic" or "chromatic."

It is a pity that more examples of Hamilton's use of the harmoniai are not extant. From this limited sample, it appears that Schlesinger's system succeeds as a "new language of music."

Schlesinger's harmoniai have inspired other composers, including Harry Partch and Cris Forster. Partch devoted a large part of his chapter on other systems of just intonation to her work, citing it as a justification to proceed on to ratios of 13 (Partch [1949] 1974). He correctly identified her harmoniai with his Utonalities, with the addition of the Secondary Ratio, 16/15. Forster has constructed several instruments embodying the ratios of 13 in a Partch tonality diamond context. He has also composed a considerable body of music for these instruments (Forster 1979).

8-7. *Chordal relations between related harmoniai (Schlesinger 1939, 543–44).*

D	ML	HL	L	HP	P	HD	D	ML
TETRACHORDAL CHORDS								
11	7	10	13	9	6	8	11	7
7	10	13	9	12	8	11	7	10
8	11	14	10	13	9	12	8	11
11	14	20	13	18	12	16	11	14
MIXED CHORDS								
7	10	13	9	6	8	11	7	
10	13	9	12	8	11	7	10	
8	11	14	10	13	9	12	8	
11	14	20	13	18	12	16	11	
INTERVALS OF RESOLUTION								
11	7	10	13	9	6	8	11	
14	10	13	9	12	8	11	14	

Extensions to Schlesinger's system

Although Schlesinger's system suffers from internal inconsistencies and omissions, her scales form a fascinating system in their own right, independent of their questionable historical status. The most obvious of the corrections or enhancements is to rationalize her enharmonic and chromatic forms so that all three forms of each harmonia are distinct. The next step is the definition of local tritai synemmenon in each of the tonoi so that correct hypo-modes and conjunct harmoniai may be constructed. Finally, new harmoniai based on modal determinants not used by Schlesinger are proposed. These new modal determinants range from 15 to 33.

Rationalization of the harmoniai

The first and most obvious extension to Schlesinger's system is to furnish distinct chromatic and enharmonic forms for her diatonic harmoniai. This may be done by katapyknosis of the diatonic with the multipliers 2 and 4.

To obtain the corrected chromatic versions, the first interval of each tetrachord of the diatonic harmoniai is linearly divided into two parts. The two new intervals are retained while simultaneously deleting the topmost note of each tetrachord to create the characteristic interval of the genus. By this process, the old diatonic first intervals become the pykna of the new chromatic forms.

The enharmonic is created analogously by katapyknosis with four. The first two new intervals are retained, leading to pykna which consist of the chromatic first intervals. This procedure is equivalent to performing katapyknosis with two on the chromatic genera resulting from the operations above.

Wilson has suggested performing katapyknosis with 3 to produce *trichromatic* forms (personal communication). Ptolemy used the same technique to generate his shades. This operation produces two forms, a 1 + 1 form in which the two lowest successive intervals are retained and a 1 + 2 form in which the lowest and the sum of the two highest are used. The pykna of the 1 + 1 and 1 + 2 forms are thus different and the 1 + 1 form tends to melodically approximate the enharmonic. A third form, the 2 + 1, potentially exists, but would violate Greek melodic canons (chapter 3).

In an analogous manner, katapyknosis by 5 and 6 are possible if the interval to be divided is large enough. These divisors generate what may be called *pentachromatic*, *pentenharmonic*, *hexachromatic*, and *hexenharmonic* genera. The forms of the rationalized harmoniai including the two trichromatic as well as the pentachromatic genera, created from a 2 + 3 division of the pyknon, are shown in 8-8.

If one generates all the forms of a harmonia which do not violate accepted melodic canons by katapyknosis with the numbers 1 through 6, nineteen genera result. The Hypermixolydian or "bastard Hypodorian" provides a good example of this process because the first diatonic interval is the comparatively large septimal tone 8/7 (231 cents). The nineteen katapyknotic genera of her "bastard Hypodorian" are shown in 8-9.

Local tritai synemmenon

Although all of the diatonic harmoniai can be represented as octave species of the Dorian harmonia (plus trite synemmenon) by choosing different notes as modal determinants, in the homonymous tonoi the central octave is occupied by the notes of the corresponding harmoniai. Since all of the tonoi are structurally as well as logically equivalent, the argument which demanded that 15 replace 14 in the hypo-modes of the Dorian requires that a local trite synemmenon be defined in each tonos. Otherwise, the

8-8. *Rationalized harmoniai. These harmoniai should be compared to Schlesinger's own as significant differences exist between these and some of hers in the chromatic and enharmonic genera. Three new genera are also provided; these are based on katapyknosis by 3 and 5 instead of 2 and 4. To avoid fractions, some numbers have been doubled. In principle, 14 may be substituted for 15 in the hypo-modes. 14 alternates with 15 in the Hypolydian. To preserve melodic contour, the chromatic and enharmonic forms of the Hypodorian are derived from the "bastard" harmonia. The forms of the lower tetrachords of Schlesinger's preferred harmonia would be 32 31 30 24, 48 47 46 36, 48 47 45 36, and 80 78 75 60..*

Mixolydian

DIATONIC

14 13 12 11 10 9 8 7

CHROMATIC

28 27 26 22 20 19 18 14

TRICHROMATIC 1

42 41 40 33 30 29 28 21

TRICHROMATIC 2

42 41 39 33 30 29 27 21

ENHARMONIC

56 55 54 44 40 39 38 28

PENTACHROMATIC

70 68 65 55 50 48 45 35

Lydian

DIATONIC

13 12 11 10 9 8 7 13

CHROMATIC

26 25 24 20 18 17 16 13

TRICHROMATIC 1

39 38 37 30 27 26 25 39

TRICHROMATIC 2

39 38 36 30 27 26 24 39

ENHARMONIC

52 51 50 40 36 35 34 26

PENTACHROMATIC

65 63 60 50 45 43 40 65

Phrygian

DIATONIC

12 11 10 9 8 7 13 6

CHROMATIC

24 23 22 18 16 15 14 12

TRICHROMATIC 1

36 35 34 27 24 23 22 18

TRICHROMATIC 2

36 35 33 27 24 23 21 18

ENHARMONIC

48 47 46 36 32 31 30 24

PENTACHROMATIC

60 58 55 45 40 38 35 30

Dorian

DIATONIC

11 10 9 8 7 13 6 11

CHROMATIC

22 21 20 16 14 27 13 11

TRICHROMATIC 1

33 32 31 24 21 41 40 33

TRICHROMATIC 2

33 32 30 24 21 20 39 33

ENHARMONIC

44 43 42 32 28 55 27 22

PENTACHROMATIC

55 53 50 40 35 34 65 55

Hypolydian

DIATONIC

10 9 8 7 13 6 11 5

CHROMATIC

20 19 18 14 13 25 12 10

TRICHROMATIC 1

30 29 28 21 39 38 37 15

TRICHROMATIC 2

30 29 27 21 39 38 36 15

ENHARMONIC

40 39 38 28 26 51 25 20

PENTACHROMATIC

50 48 45 35 65 63 30 25

Hypophrygian

DIATONIC

18 16 15 13 12 11 10 9

CHROMATIC

18 17 16 13 12 23 11 9

TRICHROMATIC 1

54 52 50 39 36 35 34 27

TRICHROMATIC 2

54 52 48 39 36 35 33 27

ENHARMONIC

36 35 34 26 24 47 23 18

PENTACHROMATIC

90 86 80 65 60 58 55 45

Hypodorian

DIATONIC

16 15 13 12 11 10 9 8

CHROMATIC

32 30 28 24 22 21 20 16

TRICHROMATIC 1

48 46 44 36 33 32 31 24

TRICHROMATIC 2

48 46 42 36 33 32 30 24

ENHARMONIC

64 62 60 48 44 43 42 32

PENTACHROMATIC

80 76 70 60 55 53 50 40

three hypo-modes in each tonos would be merely cyclic permutations of the original sequence and would therefore lack modal distinction. These tritai synemmenon are also needed to to form what Schlesinger would probably term conjunct harmoniai.

The new tritai synemmenon may be supplied by analogy through katapyknosis of the disjunctive tone by 2. These additions, of course, increase the number of possible scale forms, as the new notes may alternate with the lesser of their neighbors as 15 alternates with 14 in the Dorian prototype. This alternation generates fairly wide intervals in the range of augmented seconds and gives the harmoniai containing them a chromatic or harmonic minor flavor not present in the corresponding modes of the Dorian harmonia.

8-9. *The nineteen genera of Schlesinger's "bastard Hypodorian" harmonia. Beyond 6x the intervals are usually too small to be useful melodically. The numbers after the genus abbreviations distinguish the various species. The multiplier refers to the multiplication of the modal determinants in katapyknosis. The species are defined by the unit-proportions of their pykna. The 4x, 5x, and 6x divisions define genera with both enharmonic and chromatic melodic properties.*

NO.	DIVISION	MULTIPLIER	SPECIES
	DIATONIC		
D1	16 14 13 12 11 10 9 8	1x	1+1
	CHROMATIC		
C1	16 15 14 12 11 21 10 8	2x	1+1
	TRICHROMATIC		
T1	24 23 22 18 33 32 31 12	3x	1+1
T2	24 23 21 18 33 32 30 12	3x	1+2
	ENHARMONIC/CHROMATIC		
E1	32 31 30 24 22 43 21 16	4x	1+1
E2	32 31 29 24 22 43 41 16	4x	1+2
E3	32 31 28 24 22 43 20 16	4x	1+3
	PENTACHROMATIC/PENTENHARMONIC		
P1	40 39 38 30 55 27 53 20	5x	1+1
P2	40 39 37 30 55 27 26 20	5x	1+2
P3	40 39 36 30 55 27 51 20	5x	1+3
P4	40 39 35 30 55 27 25 20	5x	1+4
P5	40 38 36 30 55 53 51 20	5x	2+2
P6	40 38 35 30 55 53 50 20	5x	2+3
	HEXACHROMATIC/HEXENHARMONIC		
H1	48 47 46 36 33 65 32 24	6x	1+1
H2	48 47 45 36 33 65 63 24	6x	1+2
H3	48 47 44 36 33 65 62 24	6x	1+3
H4	48 47 43 36 33 65 61 24	6x	1+4
H5	48 47 42 36 33 65 30 24	6x	1+5
H6	48 46 43 36 33 64 61 24	6x	2+3

8-10. *Conjunct rationalized harmoniai. These harmoniai are formed in analogy to the conjunct Dorian of Schlesinger. The Hypodorian forms are based on the "bastard" harmonia. The lower tetrachords of Schlesinger's preferred form are 32 10 30 24, 48 47 46 36, 48 47 45 36, and 80 78 75 60.*

Mixolydian

DIATONIC

14 13 12 11 21 9 8 7

CHROMATIC

28 27 26 22 21 20 16 14

TRICHROMATIC 1

42 41 40 33 32 31 24 21

TRICHROMATIC 2

42 41 39 33 32 30 24 21

ENHARMONIC

56 55 54 44 43 42 32 28

PENTACHROMATIC

70 68 65 55 53 50 40 35

Lydian

DIATONIC

13 12 11 10 19 8 7 13

CHROMATIC

26 25 24 20 19 18 14 13

TRICHROMATIC 1

39 38 37 30 29 28 21 39

TRICHROMATIC 2

39 38 36 30 29 27 21 39

ENHARMONIC

52 51 50 40 39 38 28 26

PENTACHROMATIC

65 63 60 55 50 48 45 65

Phrygian

DIATONIC

24 22 20 18 17 14 13 6

CHROMATIC

24 23 22 18 17 16 13 12

TRICHROMATIC 1

36 35 34 27 26 25 39 18

TRICHROMATIC 2

36 35 33 54 26 24 39 18

ENHARMONIC

48 47 46 36 35 34 26 24

PENTACHROMATIC

60 58 55 45 40 38 65 30

Dorian

DIATONIC

11 10 9 8 15 13 6 11

CHROMATIC

22 21 20 16 15 14 12 11

TRICHROMATIC 1

33 32 31 24 23 22 18 33

TRICHROMATIC 2

33 32 30 24 23 21 18 33

ENHARMONIC

44 43 42 32 31 30 24 22

PENTACHROMATIC

55 53 50 40 35 33 30 55

Hypolydian

DIATONIC

20 18 16 15 13 12 11 10

CHROMATIC

20 19 18 15 14 13 11 10

TRICHROMATIC 1

60 58 56 45 43 41 33 30

TRICHROMATIC 2

60 58 54 45 43 39 33 30

40 39 38 30 29 28 22 20

PENTACHROMATIC

50 48 45 75 65 65 55 50

Hypophrygian

DIATONIC

18 16 15 13 25 11 10 9

CHROMATIC

18 17 16 13 25 12 10 9

TRICHROMATIC 1

54 52 50 39 38 37 30 27

TRICHROMATIC 2

54 52 48 39 38 36 30 27

ENHARMONIC

36 35 34 26 51 25 20 18

PENTACHROMATIC

90 86 80 65 63 60 50 45

Hypodorian

DIATONIC

16 15 13 12 23 10 9 8

CHROMATIC

32 30 28 24 23 22 18 16

TRICHROMATIC 1

48 46 44 36 35 34 27 24

TRICHROMATIC 2

48 46 42 36 35 33 27 24

ENHARMONIC

64 62 60 48 47 46 36 32

PENTACHROMATIC

80 76 70 60 58 55 45 40

8-11. *Synopsis of the rationalized tonoi. The tonoi are transpositions of the Dorian modal sequence so that the modal determinant of each harmonia falls on hypate meson. A local trite synemmenon has been defined in each of these harmoniai. In the Hypolydian, 15 alternates with 14. When mese falls on 14, trite synemmenon is 27 (27/22). The Hypodorian also has a "bastard" form which runs from proslambanomenos to mese in the Dorian tonos. The first tetrachord is 16 14 13 12.*

NAME	P	HH	HM	M	TS	P	ND
MIXOLYDIAN	44	40	28	22	21	20	14
LYDIAN	40	36	26	20	19	18	13
PHRYGIAN	36	32	24	18	17	16	12
DORIAN	32	28	22	16	15	14	11
HYPOLYDIAN	28	26	20	15/2	14	13	10
HYPOPHRYGIAN	26	24	18	13	25/2	12	9
HYPODORIAN	24	22	16	12	23/2	11	8

New conjunct forms

The new tritai synemmenon combine with the remaining tones to yield conjunct forms for each of the harmoniai. In order to preserve genera-specific melodic contours, a variation on the usual principle of construction was employed in the derivation of these scales. The procedure may be thought of as a type of inverse katapyknosis utilizing the note alternative to the local trite synemmenon in some cases. These conjunct harmoniai are listed in 8-10 in their diatonic, various chromatic, and enharmonic forms. The tuning of the principal structural notes of the rationalized tonoi is summarized in 8-11.

New modal determinants

As mentioned previously, one of the most noticeable inconsistencies in Schlesinger's system is the lack of a harmonia whose modal determinant is 15. Similarly in the new conjunct harmoniai, modal determinants of 17, 19, 21, 23, and 25 are implied by the local tritai synemmenon of the rationalized tonoi. Schlesinger herself stipulates the existence of harmoniai on 21 and 27 as later modifications of the Dorian and Lydian harmoniai. She claimed that these harmoniai were created by shifting their modal determinants one degree lower.

Additional harmoniai on modal determinants 29 and 31 may be added without exceeding the bounds of the Perfect Immutable System. To these may be added a harmonia on 33, which, though it exceeds the boundaries of the Dorian tonos, is included in the ranges of the tonoi of 8-12 and 8-13. The normal or disjunct forms of these new harmoniai are shown in 8-12 and the conjunct, which use their local tritai synemmenon, in 8-13. A summary of these new harmoniai is given in 8-14.

8-12 (next page). *New harmoniai. These harmoniai were created to fill in the gaps in Schlesinger's system, although some, such as tonoi-15, -21, and -27, are implied in her text. Three new genera are also provided; these are based on katapyknosis by 3 and 5 instead of 2 and 4. In principle, 14 may be substituted for 15 in these harmonia, save for tonos-15 where the Mixolydian harmonia would result. Similarly, 21 may replace 22 and 27, 26, except when doing so would change the modal determinant. In the diatonic genus when the first interval above the modal determinant is roughly a semitone, chromatic alternation with the next highest degree would be melodically acceptable.*

Tonos-15

DIATONIC

15 13 12 11 10 9 8 15

CHROMATIC

15 14 13 11 10 19 9 15

TRICHROMATIC 1

45 44 43 33 30 29 28 45

TRICHROMATIC 2

45 44 42 33 30 29 27 45

ENHARMONIC

30 29 28 22 20 39 19 15

PENTACHROMATIC

75 71 65 55 50 48 45 75

Tonos-17

DIATONIC

17 15 13 12 11 10 9 17

CHROMATIC

17 16 15 12 11 21 10 17

TRICHROMATIC 1

51 49 47 36 33 32 31 51

TRICHROMATIC 2

51 49 45 36 33 32 30 51

ENHARMONIC

34 33 32 24 22 43 21 17

PENTACHROMATIC

85 81 75 60 55 53 50 85

Tonos-19

DIATONIC

19 18 16 14 13 12 11 19

CHROMATIC

19 18 17 14 13 25 12 19

TRICHROMATIC 1

57 55 53 42 39 38 37 57

TRICHROMATIC 2

57 55 51 42 39 38 36 57

ENHARMONIC

38 37 36 28 26 51 25 19

PENTACHROMATIC

95 91 85 70 65 63 60 95

Tonos-21

DIATONIC

21 19 18 16 14 13 12 21

CHROMATIC

21 20 19 16 14 27 13 21

TRICHROMATIC 1

63 61 59 48 42 41 40 63

TRICHROMATIC 2

63 61 57 48 42 41 39 63

ENHARMONIC

42 41 40 32 28 55 27 21

PENTACHROMATIC

105 101 95 80 70 68 65 105

Tonos-23

DIATONIC

23 21 20 18 16 14 13 23

CHROMATIC

23 22 21 18 16 15 14 23

TRICHROMATIC 1

69 67 65 54 48 46 44 69

TRICHROMATIC 2

69 67 63 54 48 46 42 69

ENHARMONIC

46 45 44 36 32 31 30 23

PENTACHROMATIC

115 111 105 90 80 76 70 115

Tonos-25

DIATONIC

25 22 20 18 16 14 13 25

CHROMATIC

50 47 22 36 32 30 28 25

TRICHROMATIC 1

75 72 69 54 48 46 44 75

TRICHROMATIC 2

75 72 66 54 48 46 42 75

ENHARMONIC

50 97 47 36 32 31 30 25

PENTACHROMATIC

125 119 110 90 80 76 70 125

Tonos-27

DIATONIC

27 24 21 20 18 16 14 27

CHROMATIC

54 51 48 40 36 34 32 27

TRICHROMATIC 1

81 78 75 60 54 52 50 81

TRICHROMATIC 2

81 78 72 60 54 52 48 81

ENHARMONIC

101 102 40 36 35 34 54

PENTACHROMATIC

135 129 120 100 90 86 80 135

Tonos-29

DIATONIC

29 26 24 22 20 18 16 29

CHROMATIC

29 28 27 22 20 19 18 29

TRICHROMATIC 1

87 85 83 66 60 58 56 87

TRICHROMATIC 2

87 85 81 66 60 58 54 87

ENHARMONIC

58 57 56 44 40 39 38 29

PENTACHROMATIC

145 141 135 110 100 96 90 145

Tonos-31

DIATONIC

31 28 26 23 22 20 18 31

CHROMATIC

31 29 27 23 22 21 20 31

TRICHROMATIC 1

93 89 85 69 66 64 62 93

TRICHROMATIC 2

93 89 81 69 66 64 60 93

ENHARMONIC

31 30 29 23 22 43 21 31

PENTACHROMATIC

155 147 135 115 110 106 100 155

Tonos-33

DIATONIC

33 30 27 24 22 2018 33

CHROMATIC

33 31 29 24 22 21 20 33

TRICHROMATIC 1

99 96 93 72 66 64 62 99

TRICHROMATIC 2

99 96 90 72 66 64 60 99

ENHARMONIC

33 32 31 24 22 43 21 33

PENTACHROMATIC

165 159 150 120 110 106 100 165

Tonos-21: Schlesinger claimed that the Dorian 22 was lowered in the PIS to 21 and that of the Lydian from 27 to 26; tonos-21 is thus the Dorian of the PIS. Tonos-25: It has proven difficult to obtain harmoniai whose melodic forms are characteristic of the genera. This tonos demands chromatic alternatives (17 for 16, 48 for 47, 23 for 22, 97 for 98, etc.). Tonos-27: This was conjectured by Schlesinger to be the Syntonolydian. Note 21 may alternate with 22. It may be described as the Lydian of the PIS. Alternative forms are 27 24 22 20 18 16 14 27, 27 26 25 20 18 17 16 27, and 54 53 52 40 36 35 34 27. Tonos-29: In the diatonic, 26 may alternate with 27. Tonos-31: These harmoniai admit several variants where 24 and 23, 29 and 30, 28 and 27 are alternatives. In tonos-33, the diatonic has a variant 33 29 27 24, the chromatic 33 63 30 24, the first trichromatic 99 95 91 72, the second trichromatic 99 95 87 72, and the pentachromatic 165 157 145 120 110.

Tonos-15

DIATONIC

15 13 12 11 21 18 16 15

CHROMATIC

15 14 13 11 21 20 16 15

TRICHROMATIC 1

45 44 43 33 32 31 24 45

TRICHROMATIC 2

45 44 42 33 32 30 24 45

ENHARMONIC

30 29 28 22 43 21 16 15

PENTACHROMATIC

75 71 65 55 53 50 40 75

Tonos-17

DIATONIC

17 15 13 12 23 10 9 17

CHROMATIC

17 16 15 12 23 11 9 17

TRICHROMATIC 1

51 49 47 36 35 34 27 51

TRICHROMATIC 2

51 49 45 36 35 33 27 51

ENHARMONIC

34 33 32 24 47 23 18 17

PENTACHROMATIC

85 81 75 60 58 55 90 85

Tonos-19

DIATONIC

19 18 16 14 27 12 11 19

CHROMATIC

19 18 17 14 27 13 11 19

TRICHROMATIC 1

57 55 53 42 41 40 33 57

TRICHROMATIC 2

57 55 51 42 41 39 33 57

ENHARMONIC

38 37 36 28 55 54 22 19

PENTACHROMATIC

95 91 85 70 68 65 55 95

Tonos-21

DIATONIC

21 19 18 16 15 13 12 21

CHROMATIC

21 20 19 16 15 14 12 21

TRICHROMATIC 1

63 61 59 48 46 44 36 63

TRICHROMATIC 2

63 61 57 48 46 42 36 63

ENHARMONIC

42 41 40 32 31 30 24 21

PENTACHROMATIC

105 101 95 80 76 70 60 105

Tonos-23

DIATONIC

23 21 20 18 17 14 13 23

CHROMATIC

23 22 21 18 17 16 13 23

TRICHROMATIC 1

69 67 65 54 52 50 39 69

TRICHROMATIC 2

69 67 63 54 52 48 39 69

ENHARMONIC

46 45 44 36 35 34 26 23

PENTACHROMATIC

115 111 105 90 86 80 65 115

Tonos-25

DIATONIC

25 22 20 18 17 14 13 25

CHROMATIC

50 47 44 36 34 32 26 25

TRICHROMATIC 1

75 72 69 54 52 50 39 75

TRICHROMATIC 2

75 72 66 54 52 48 39 75

ENHARMONIC

50 97 47 36 35 34 36 25

PENTACHROMATIC

125 119 110 90 86 80 65 125

Tonos-27

DIATONIC

27 24 21 20 19 16 14 27

CHROMATIC

54 51 48 40 38 36 28 27

TRICHROMATIC 1

81 78 75 60 58 56 42 81

TRICHROMATIC 2

81 78 72 60 58 54 42 81

ENHARMONIC

54 105 51 40 39 38 28 27

PENTACHROMATIC

135 129 120 100 96 90 70 135

Tonos-29

DIATONIC

29 26 24 22 21 18 16 29

CHROMATIC

29 28 27 22 21 20 16 29

TRICHROMATIC 1

87 85 83 66 64 62 48 87

TRICHROMATIC 2

87 85 81 66 64 60 48 87

ENHARMONIC

58 57 56 44 43 42 32 29

PENTACHROMATIC

145 141 135 110 106 100 80 145

Tonos-31

DIATONIC

31 28 26 23 22 20 18 31

CHROMATIC

31 29 27 23 22 21 18 31

TRICHROMATIC 1

93 89 85 69 67 65 54 93

TRICHROMATIC 2

93 89 81 69 67 63 54 93

ENHARMONIC

31 30 29 23 45 44 36 31

PENTACHROMATIC

155 147 135 115 111 105 90 155

Tonos-33

DIATONIC

33 30 27 24 23 20 18 33

CHROMATIC

33 31 29 24 23 22 18 33

TRICHROMATIC 1

99 96 93 72 70 68 54 99

TRICHROMATIC 2

99 96 90 72 70 66 54 99

ENHARMONIC

33 32 31 24 47 46 18 33

PENTACHROMATIC

165 159 150 120 116 110 90 165

8-13. *New conjunct harmoniai. In this context, conjunct means employing the local tonos-specific trite synemmenon.*

8-14. *Synopsis of the new tonoi. The tonoi are transpositions of the Dorian modal sequence so that the determinant of each harmonia falls on hypate meson. A local trite synemmenon for each of the harmoniai has been defined. Certain odd or prime number modal determinants have been expressed as fractions, i.e. 21/2, to indicate the higher octave since the modal determinants represent aliquot parts of vibrating air columns or strings. Modal determinants 14 (28) and 15 (30) are alternates. Tonos-31: in the conjunct form, mese is 23, trite synemmenon is 22.*

	P	HH	HM	M	TS	P	ND
TONOS-15	22	20	15	11	21/2	10	15/2
TONOS-17	24	22	17	12	23/2	11	17/2
TONOS-19	28	26	19	14	27/2	13	19/2
TONOS-21	32	28	21	16	15	14	21/2
TONOS-23	36	32	23	18	17	16	23/2
TONOS-25	36	32	25	18	17	16	25/2
TONOS-27	40	36	27	20	19	18	27/2
TONOS-29	44	40	29	22	21	20	29/2
TONOS-31	48	44	31	24	22	22	31/2
TONOS-33	48	44	33	24	23	22	33/2

Harmonizing the new harmoniai

The new harmoniai may be harmonized by methods analogous to those Elsie Hamilton employed with Schlesinger's diatonic harmoniai. The tetrachordal framework chords of both the disjunct and conjunct forms of the new harmoniai are shown in 8-15.

The framework chords from the new conjunct forms are particularly interesting harmonically as they provide a means of incorporating the new harmoniai with the older system. Because many of the modal determinants of the new harmonia are prime numbers, their tetrachordal framework chords do not share many notes with the ones from the older scales. Certain chords, however, from the new conjunct harmoniai do share notes with the framework chords of the older forms and thus allow one to modulate by common tone progressions. These chords may also be used in progressions similar to those in 8-6c and 8-7.

Moreover, these chords may be used to harmonize the mesopykna of the chromatic harmoniai and the oxypykna of the enharmonic which seemingly lay outside of Hamilton's harmonic concerns.

Harmoniai with more than seven tones

Although it is quite feasible to define harmoniai with modal determinants between 33 and 44 (the limit of the Mixolydian tonos), it becomes increasingly difficult to decide the canonical forms such harmoniai might take because of the rapidly increasing number of chromatic or alternative tones available in the octave.

Rather than omit the extra tones in these and the harmoniai with smaller modal determinants, one may define harmoniai with more than seven tones and utilize the resulting melodic and harmonic resources.

8-15. *Harmonization of the new harmoniai. Tetrachordal framework chords.*

	DISJUNCT	CONJUNCT
HARMONIA-15	15:11:10:15/2	15:11:8:15/2
HARMONIA-17	17:12:11:17/2	17:12:9:17/2
HARMONIA-19	19:14:13:19/2	19:14:11:19/2
HARMONIA-21	21:16:14:21/2	21:16:12:21/2
HARMONIA-23	23:18:16:23/2	23:18:13:23/2
HARMONIA-25	25:18:16:25/2	25:18:13:25/2
HARMONIA-27	27:20:18:27/2	27:20:14:27/2
HARMONIA-29	29:22:20:29/2	29:22:16:29/2
HARMONIA-31	31:24:22:31/2, 31:23:22:31/2	31:23:18:31/2, 31:24:18:31/2
HARMONIA-33	33:24:22:33/2	33:24:18:33/2

8-16. *Harmonic forms of the Phrygian harmonia. For each of the diatonic harmoniai, the harmonic forms are obtained by taking the 2/1 complement of each ratio or interval.*

FIRST VERSION OF THE INVERTED PHRYGIAN

DIATONIC

12 13 14 16 18 20 22 24

CHROMATIC

12 14 15 16 18 22 23 24

ENHARMONIC

24 30 31 32 36 46 47 48

SECOND VERSION OF THE INVERTED PHRYGIAN

CHROMATIC

24 25 26 32 36 38 40 48

ENHARMONIC

48 49 50 64 72 74 76 96

8-17. *Harmonic forms of the conjunct Phrygian harmonia. For each of the conjunct diatonic harmoniai, the harmonic form is obtained by taking the 2/1 complement of each ratio or interval.*

FIRST VERSION OF THE INVERTED CONJUNCT PHRYGIAN HARMONIAI

DIATONIC

12 13 14 17 18 20 22 24

CHROMATIC

12 13 16 17 18 22 23 24

ENHARMONIC

24 26 34 35 36 46 47 48

SECOND VERSION OF THE INVERTED CONJUNCT PHRYGIAN HARMONIAI

CHROMATIC

24 26 27 28 36 38 40 48

ENHARMONIC

48 52 53 54 72 74 76 96

Another source of new harmoniai has been suggested by Wilson. One might insert pykna above notes other than the first and fourth degrees of the basic diatonic modal sequence. Interesting variations may also be discovered by inserting more than two pykna, or any number at any location. The final result of this procedure is to generate "close-packed" scales with many more than seven notes.

Harmonic forms of the harmoniai

Schlesinger's original harmoniai and all of the new scales generated in analogy with hers are 1- or 2-octave sections of the subharmonic series. These musical structures may be converted to sections of the harmonic series by replacing each of their tones with their 2/1 complements or octave inversions.

The resulting harmonic forms may be used in exactly the same way as the originals, save that the modalities of the chords (major or minor) and the melodic contours of the scales are reversed, i.e., the intervals become smaller rather than larger as one ascends from the lowest tone.

In general, chords from the harmonic series are more consonant than those from the subharmonic. However, the tones of the harmonic scales are more likely to be heard as arpeggiated chords than are the scalar tones of the subharmonic forms.

There is only one form of each of the inverted diatonic harmoniai, but the chromatic, enharmonic and other katapyknotic forms (8-9) have two versions. The first forms are the octave complements of the corresponding subharmonic originals and these forms have their pykna at the upper end of each tetrachord. The second versions are produced by dividing the initial intervals of the two tetrachords of the inverted diatonic forms as in the generation of the chromatic and other katapyknotic forms of 8-9. An example which illustrates these operations is shown in 8-16. The Phrygian harmonia, of modal determinant 12, is inverted and then divided to yield the diatonic, chromatic and enharmonic forms. Both versions of the chromatic and enharmonic harmoniai are listed, and the other katapyknotic forms may be obtained by analogy.

Conversely, the second of the new harmonic forms may be inverted to derive new subharmonic harmoniai whose divided pykna lie at the top of their tetrachords. These too are listed in 8-16.

Conjunct harmoniai may also be inverted to generate harmonic

8-18. *Wilson's diaphonic cycles. These diaphonic cycles (diacycles) may be constructed on sets of strings tuned alternately a 3/2 and 4/3 apart since the largest divided interval is the 3/2. The order of the segments, nodes, and conjunctions may be permuted according to the following scheme: a/b · c/d = a/d · c/b = 2/1 and c/d · a/b = c/b · a/d = 2/1. Alternative conjunctions are indicated by primed nodes, i.e. c′, d′. Some diacycles such as number 21 have two independent sets of nodes and conjunctions. The second is symbolized by e f g h.*

1. 9 8 7 6
a c b, d
(3/2 · 4/3)

2. 12 11 10 9 8
a, c d b
(3/2 · 4/3)

3. 18 17 16 15 14 13 12
a c b, d
(3/2 · 4/3)

4. 21 20 19 18 17 16 15 14
a c d b
(3/2 · 4/3; 10/7 · 7/5)

5. 24 23 22 21 20 19 18 17 16
a, c b d
(3/2 · 4/3)

6. 27 26 25 24 23 22 21 20 19 18
a c b, d
(3/2 · 4/3)

7. 30..........28..........21 20
a c d b
(3/2 · 4/3; 10/7 · 7/5)

8. 33 32..........24..........22
a c d b
(3/2 · 4/3; 16/11 · 11/8)

9. 36..........32..........27..........24
a, c c′ d b
(3/2 · 4/3)

10. 39..........36..........27 26
a c d b
(3/2 · 4/3; 13/9 · 18/13)

11. 42..........40..........30..........28
a c d b
(3/2 · 4/3; 10/7 · 7/5)

12. 45 44..........40..........33..........30
a c′ c d′ b, d
(3/2 · 4/3; 22/15 · 15/11)

13. 48..........44..........36..........33 32
a, c′ c d′ d b
(3/2 · 4/3; 16/11 · 11/8)

14. 51..........48..........36..........34
a c d b
(3/2 · 4/3; 17/12 · 24/17)

15. 54..........52..........48..........39..........36
a c′ c d′ b, d
(3/2 · 4/3; 13/9 · 18/13)

16. 57 56..........52..........42..........39 38
a c′ c d′ d b
(3/2 · 4/3; 19/14 · 28/19; 19/13 · 26/19)

17. 60..........56..........42..........40
a c d b
(3/2 · 4/3; 10/7 · 7/5)

18. 63..........60..........56..........45..........42
a c′ c d′ b, d
(3/2 · 4/3; 10/7 · 7/5)

19. 66..........64..........60..........48..........45 44
a c′ c d′ d b
(3/2 · 4/3; 22/15 · 15/11; 16/11 · 11/8)

20. 69 68..........64..........51..........48..........46
a c′ c d′ d b
(3/2 · 4/3; 23/16 · 32/23; 23/17 · 34/23)

21. 72..........70..........68..........64..........51 50 49 48
a e, g c′ c d′ h f b, d
(3/2 · 4/3; 10/7 · 7/5; 24/17 · 17/12)

22. 75..........68..........51 50
a c d b
(3/2 · 4/3; 25/17 · 34/25)

23. 78..........76..........57..........52
a c d b
(3/2 · 4/3; 26/19 · 19/13)

24. 81 80..........77..........60..........56 55 54
a c, e g d h f b

8-19. *Diacycles on 20/13. These diacycles can be constructed on strings 13/10 and 20/13 apart.*

40 39 ... 36 .... 30 ... 27 26
a c,e g d f b,h
(20/13 · 13/10; 3/2 · 4/3; 13/9 · 18/13)

60 ... 56 ... 52 .... 42 .. 40 39
a,e g c f,h d b
(20/13 · 13/10; 3/2 · 4/3; 10/7 · 7/5)

80 .. 78 ... 76 .... 60 ... 57 52
a c,e g d f b,h
(20/13 · 13/10; 3/2 · 4/3; 26/19 · 19/13)

100 99 ... 96 .... 91 .... 72 ... 70 ..... 66 65
a e g c h d f b
(20/13 · 13/10; 10/7 · 7/5; 3/2 · 4/3; 16/11 · 11/8)

8-20. *Triaphonic and tetraphonic cycles on 4/3 and 5/4. (1) may be constructed on three strings tuned to 1/1, 4/3, and 3/2. (2) requires strings tuned to 1/1, 4/3, and 3/2. (3) may be realized on four strings tuned to 1/1, 6/5, 147/100 and 42/25.*

20 19 18 17 16 15
a, c e d b, f
(4/3 · 5/4 · 6/5)

28 27 .... 24 21
a, c e d b, f
(4/3 · 7/6 · 9/7)

50 49 48 .... 42 40
a e c, g f, h b, d
(5/4 · 6/5 · 7/6 · 8/7)

forms as shown in 8-17. In this case, the disjunctive tone is at the bottom with the two tetrachords linked by conjunction above.

These operations may be applied to all of the harmoniai described above. Similarly, the other musical structures presented in the remainder of this chapter may also be inverted.

Other directions: Wilson's diaphonic cycles

Ervin Wilson has developed a set of scales, the *diaphonic cycles,* which combine the repeated modular structure of tetrachordal scales with the linear division of Schlesinger's harmoniai (Wilson, personal communication).

The diaphonic cycles, or less formally *diacycles,* may be understood most easily by examining the construction of the two simplest members in 8-18.

In diacycle 1, the interval 3/2, which is bounded by the nodes *a* and *b,* is divided linearly to generate the subharmonic sequence 9 8 7 6 or 1/1 9/8 9/7 3/2. Subtended by this 3/2 is the linearly divided 4/3 bounded by the nodes *c* and *d.* This segment forms the sequence 8 7 6 or 1/1 8/7 4/3. Five-tone scales may be produced by joining these two melodic segments with a common tone to yield 1/1 9/8 9/7 3/2 12/7 2/1 (*a – b* on 1/1, then *c–d* on 3/2) and 1/1 8/7 4/3 3/2 12/7 2/1 (*c–d* on 1/1, then *a–b* on 4/3):

9 8 7 (6) and 8 7 (6)
(8) 7 6 (9) 8 7 6

The tones in parentheses are common to the two segments.

Diaphonic cycle 2 generates two heptatonic scales which are modes of Ptolemy's equable diatonic genus: 1/1 12/11 6/5 4/3 16/11 8/5 16/9 2/1 and 1/1 12/11 6/5 4/3 3/2 18/11 9/5 2/1. The two forms are respectively termed the conjunctive and disjunctive or tetrachordal form.

As the linear division becomes finer, scales with increasing numbers of tones are generated. At number 4, a new phenomenon emerges: the existence of another set of segments whose conjunction produces complete scales. The nodes *a,d* and *c,b* define a pair of diaphonic cycles whose segments are 10/7 and 7/5.

These diaphonic cycles can be implemented on instruments such as guitars by tuning the intervals between the strings to a succession of 3/2's and 4/3's. The fingerboards must be refretted so that the frets occur at equal aliquot parts of the string length. Wilson constructed several such guitars in the early 1960s.

8-21. *Divisions of the fifth. (1) is described as an "aulos-scale (Phrygian, reconstructed by KS)" in Schlesinger 1933. (2) is another "aulos-scale (Hypodorian)," identified with another unnamed scale of Aristoxenos (Meibomius 1652, 72). (3) is an "aulos-scale (Mixolydian)," identified with another unnamed scale of Aristoxenos. (4) is identified with yet another scale of Aristoxenos. (5) spans an augmented fifth and appears also in her interpretation of the spondeion. (6) is the "singular major" of Safiyu-d-Din (D'Erlanger 1938, 281). The Islamic genera are from Rouanet 1922. (8), Isfahan, spans only the 4/3. (9) is labeled "Zirafkend Bouzourk." Rouanet's last genus is identical to Safiyu-d-Din's scale of the same name.*

SCHLESINGER'S DIVISIONS

1. 24/23 · 23/22 · 11/9 · 9/8
2. 16/15 · 15/14 · 7/6 · 9/8
3. 28/27 · 9/8 · 8/7 · 9/8
4. 21/20 · 10/9 · 9/8 · 8/7
5. 11/10 · 10/9 · 9/8 · 8/7

ISLAMIC GENERA

6. 14/13 · 8/7 · 13/12 · 14/13 · 117/112
7. 13/12 · 14/13 · 13/12 · 287/272
8. 13/12 · 14/13 · 15/14 · 16/15
9. 14/13 · 13/12 · 36/35 · 9/8 · 10/9

Wilson has also developed a set of simpler scales on the same principles under the general name of "Helix Song." They consist of notes selected from the harmonic series on the tones 1/1 and 4/3. These have been used as the basis of a composition by David Rosenthal (Rosenthal 1979).

Triacycles and tetracycles

For the sake of completeness, some new diacycles have been constructed on the interval pair 20/13 and 13/10. These are listed in 8-19. As 20/13 is slightly larger than 3/2, some new diacycles on 3/2 are generated incidentally too.

Larger intervals and their octave complements might be used, but the increased inequality in the sizes of the two segments would probably be melodically unsatisfactory. This asymmetry may be hidden by defining three or four segments instead of merely two. A few experimental three- and four-part structures, which may be called *triacycles* and *tetracycles*, are shown in 8-20.

Linear division of the fifth

As a final note, it must be mentioned that both Schlesinger (1933) and the Islamic theorists also recognized scales derived by linear division of the fifth instead of the fourth or octave (8-21). Not surprisingly, Schlesinger's are presented as support for the authenticity of her harmoniai.

It is likely that the Islamic forms had origins that are independent of the Greek theoretical system. The genus from Safiyu-d-Din (D'Erlanger 1938) may be rationalized as being derived from the permuted tetrachord, 14/13 · 8/7 · 13/12, by dividing the disjunctive tone, 9/8, of the octave scale into two unequal parts, 14/13 and 117/112. Characteristically, all 24 permutations of the intervals were tabulated.

Rouanet's scales deviate even more from Greek models, though the tetrachordal relationship may still be seen (Rouanet 1922).